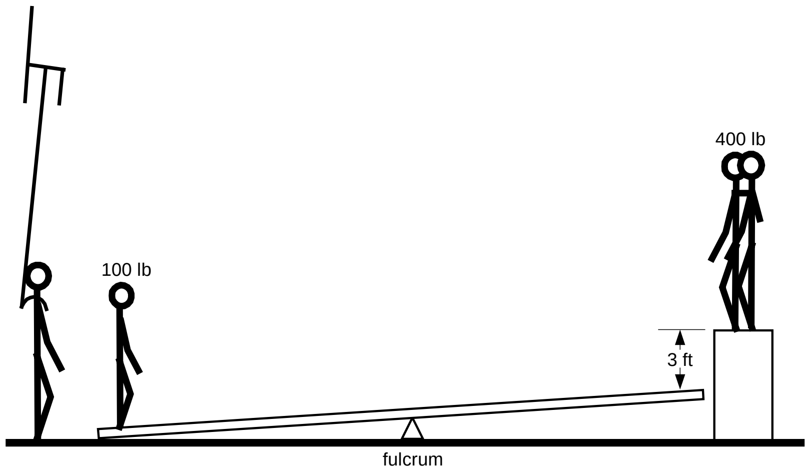
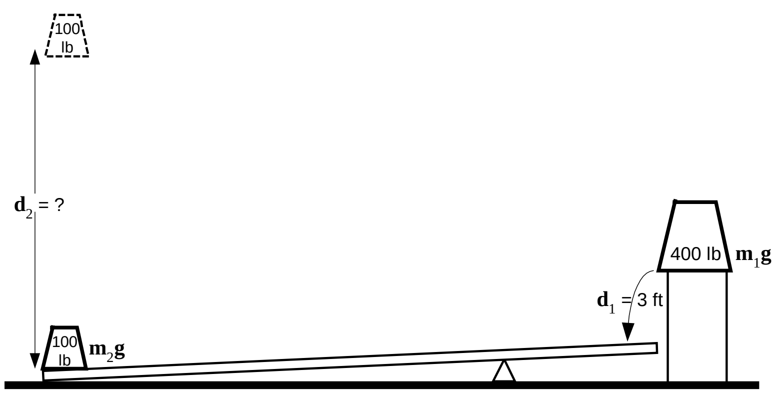
Chapter 6
Kinetic-Energy Transformers
(Teeter-Totters)

Problem



Two circus performers, each weighing 200 lb jump off a platform 3 ft above a teeter-totter and propel a 100-lb performer into the air, where he lands in a chair supported by a fourth performer. How high can the 100-lb performer go? Where should the fulcrum of the teeter-totter be placed so all the energy of the falling 400 lb is transferred to the 100-lb guy?

How High?



We'll represent the performers by idealized weights. The falling weight is mass $\mathbf{m}_1$ with weight

$$\mathbf{m}_1\mathbf{g} = 400 \text{ lb},$$

where $\mathbf{g}$ is the acceleration due to gravity $\mathbf{g} = 32$ ft/s^2. Then $\mathbf{m}_1 = (400 \text{ lb})/(32 \text{ ft/s}^2) =$ 12.5 slugs. Similarly, the weight

$$\mathbf{m}_2\mathbf{g} = 100 \text{ lb}$$

to be thrown in the air (on the left) has mass $\mathbf{m}_2$ = (100 lb)/(32 ft/s^2) = 3.125 slugs.

Mass $\mathbf{m}_1$ will fall a distance $\mathbf{d}_1$ = 3 ft. Therefore its potential energy (See Lesson 3) is

$$\mathbf{E}_{P1} = \mathbf{m}_1\mathbf{g}\cdot\mathbf{d}_1 = (400 \text{ lb})(3 \text{ ft}) = 1200 \text{ ft-lb.}$$

If all this energy is transferred to mass $\mathbf{m}_2$, then it ends up (at the top of its rise) with the same potential energy: $\mathbf{E}_{P2}$ = 1200 ft-lb. But

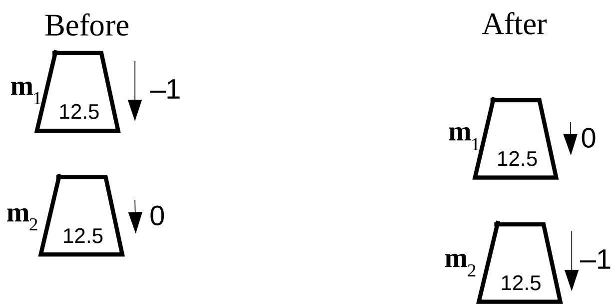
$$\mathbf{E}_{P2} = \mathbf{m}_2\mathbf{g}\cdot\mathbf{d}_2 = (100 \text{ lb})\mathbf{d}_2 = 1200 \text{ ft-lb.}$$

Therefore $$\mathbf{d}_2 = (1200 \text{ ft-lb})/(100 \text{ lb}) = 12 \text{ ft.}$$

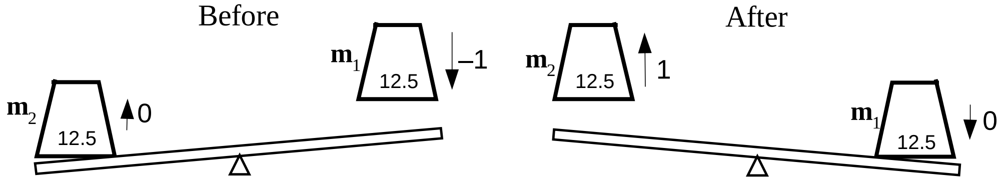
Total Energy Transfer

Where should the fulcrum of the teeter-totter be placed so mass $\mathbf{m}_1$ loses all its energy (comes to a stop) and transfers it all to $\mathbf{m}_2$? We've seen total transfer of energy before in Lesson 1, where one moving ball hits another at rest, and after the collision the first ball is at rest, and the second ball is moving. This happened only when the two balls had the same mass.

Imagine an experiment in outer space so there's no gravity. Mass $\mathbf{m}_1$ with velocity –1 is heading for an equal mass $\mathbf{m}_2$, which is standing still. (Since the masses have no weight in outer space, we refer to them being 12.5 slugs rather than 400 lb.) After the collision, $\mathbf{m}_1$ is standing still, and $\mathbf{m}_2$ has a velocity of –1. All the energy of $\mathbf{m}_1$ has been transferred to $\mathbf{m}_2$!

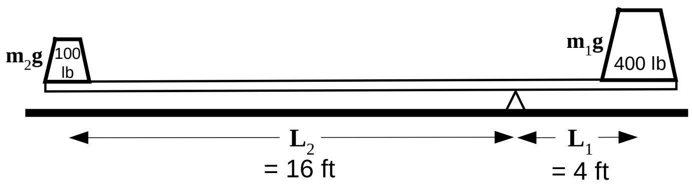


A teeter-totter can be used to reverse the direction of the energy. (The fulcrum is held fixed somehow.) If the two arms of the teeter-totter are equal length, then $\mathbf{m}_1$ will still "feel" that it is colliding with an equal mass. Again, all the energy of $\mathbf{m}_1$ is transferred to $\mathbf{m}_2$.

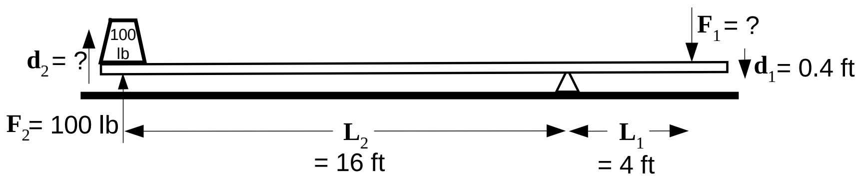


First Try at Fulcrum Position

If the two weights are not equal ($\mathbf{m}_1\mathbf{g}$ = 400 lb, and $\mathbf{m}_2\mathbf{g}$ = 100 lb), what should the ratio of the teeter-totter arms be so $\mathbf{m}_1$ still "feels" an equal mass? We know that the teeter-totter will balance if the left arm has length $\mathbf{L}_2$ = 16 ft, and the right arm has length $\mathbf{L}_1$ = 4.



We can show this by removing $\mathbf{m}_1$ and finding the force $\mathbf{F}_1$ that will support $\mathbf{m}_2$.



Let $\mathbf{F}_1$ push the right side down by a distance $\mathbf{d}_1$ = 0.4 ft, causing the 100-lb weight to go up by a distance $\mathbf{d}_2$. Since the distances $\mathbf{d}_2$ and $\mathbf{d}_1$ must have the same ratio as $\mathbf{L}_2$ and $\mathbf{L}_1$ have, then

$$\mathbf{d}_2 = 4 \cdot \mathbf{d}_1 = 4 \cdot 0.4 \text{ ft} = 1.6 \text{ ft}.$$

The lifting force $\mathbf{F}_2$ on the left is 100 lb, so the work done on the left is

$$\mathbf{W}_2 = \mathbf{F}_2\mathbf{d}_2 = (100 \text{ lb})(1.6 \text{ ft}) = 160 \text{ ft-lb}.$$

Then the work done on the right must also be 160 ft-lb.

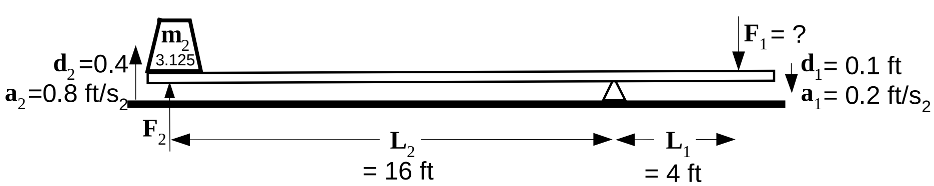
$$\mathbf{W}_1 = \mathbf{F}_1\mathbf{d}_1 = \mathbf{F}_1 \cdot (0.4 \text{ ft}) = 160 \text{ ft-lb}.$$

Therefore $$\mathbf{F}_1 = (160 \text{ ft-lb})/(0.4 \text{ ft}) = 400 \text{ lb},$$

and 400 lb on the right will balance with 100 lb on the left if the teeter-totter arms have the ratio $\mathbf{L}_2/\mathbf{L}_1 = 4$.

But we aren't interested in having $\mathbf{F}_1$ *balance* the 100-lb weight. We want $\mathbf{F}_1$ to *accelerate* the 100-lb weight. Now, the acceleration of gravity is the same on the left and on the right: $\mathbf{g}$ = 32 ft/s^2. But when $\mathbf{F}_1$ accelerates the *right* end of the teeter-totter, the *left* end will have a different acceleration.

Let's go back to outer space so gravity isn't an issue. (We use the mass $\mathbf{m}_2$ = 3.125 slugs for the 100-lb weight.) As $\mathbf{F}_1$ pushes through a distance $\mathbf{d}_1$, we'll pay attention to the



acceleration $\mathbf{a}_1$ there. Say $\mathbf{a}_1$ = 0.2 ft/s^2. If we accelerate for $\mathbf{T}$ = 1 sec, then the final velocity will be $\mathbf{v}_{final} = \mathbf{a}_1\mathbf{T}$ = (0.2 ft/s^2)(1 sec) = 0.2 ft/s. The average velocity will be half of this: $\mathbf{v}_{ave} = \frac{1}{2}\mathbf{v}_{final}$ = 0.1 ft/s, and the distance gone in that time $\mathbf{T}$ is $\mathbf{d}_1 = \mathbf{v}_{ave}\mathbf{T}$ = 0.1 ft.

Both the distance and the acceleration on the left will be greater by a factor $\mathbf{L}_2/\mathbf{L}_1 = 4$.

That is, $\mathbf{d}_2 = 4\cdot \mathbf{d}_1 = 4\cdot 0.1 \text{ ft} = 0.4 \text{ ft}$,

and $\mathbf{a}_2 = 4\cdot \mathbf{a}_1 = 4\cdot 0.2 \text{ ft/s}^2 = 0.8 \text{ ft/s}^2$.

What force $\mathbf{F}_2$ will it take to accelerate $\mathbf{m}_2$ at 0.8 ft/s^2?

$$\mathbf{F}_2 = \mathbf{m}_2\mathbf{a}_2 = (3.125 \text{ slugs})(0.8 \text{ ft/s}^2) = 2.5 \text{ lb}.$$

What force $\mathbf{F}_1$ on the right will it take to produce $\mathbf{F}_2$ = 2.5 lb on the left? The ratio $\mathbf{F}_1/\mathbf{F}_2$ is the same as the ratio $\mathbf{L}_2/\mathbf{L}_1 = 4$. Therefore

$$\mathbf{F}_1 = 4\cdot\mathbf{F}_2 = 4\cdot(2.5 \text{ lb}) = 10 \text{ lb}.$$

Now we can calculate how much mass the force $\mathbf{F}_1$ "feels." Since $\mathbf{F} = \mathbf{ma}$, the mass felt at the right is

$$\mathbf{m}_{felt} = \mathbf{F}_1/\mathbf{a}_1 = (10 \text{ lb})/(0.2 \text{ ft/s}^2) = 50 \text{ slugs}.$$

[This mass, if it were a physical thing, would have a weight $\mathbf{m}_{felt}\mathbf{g} = (50 \text{ slugs})(32 \text{ ft/s}^2) =$ 1600 lb!]

But for all the energy to be transferred, we wanted $\mathbf{m}_{felt}$ to equal the $\mathbf{m}_1$ = 12.5 slugs that falls. Instead we got $\mathbf{m}_{felt} = 4\cdot \mathbf{m}_1$. What happened? Well, the ratio $\mathbf{L}_2/\mathbf{L}_1 = 4$ became a factor twice in determining $\mathbf{m}_{felt}$, so we got a factor of $4^2 = 16$ rather that the desired factor of 4. If we go through the calculations above, we find that

$$\mathbf{m}_{felt} = (\mathbf{L}_2/\mathbf{L}_1)^2\cdot\mathbf{m}_2 = 4^2\cdot(3.125 \text{ slugs}) = 50 \text{ slugs}.$$

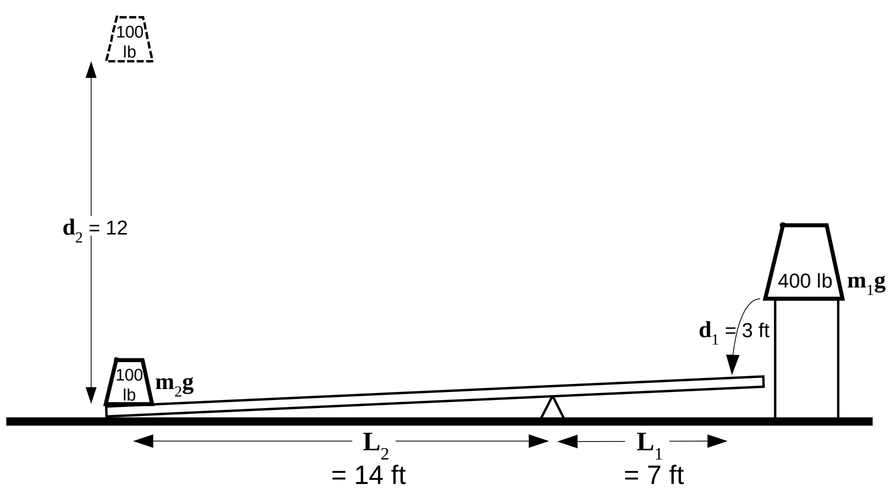
Second Try at Fulcrum Position

In order for $\mathbf{m}_{felt}$ to equal $\mathbf{m}_1$, we must choose

$$(\mathbf{L}_2/\mathbf{L}_1)^2 = \mathbf{m}_{felt}/\mathbf{m}_2 = \mathbf{m}_1/\mathbf{m}_2 = 4,$$

and $\mathbf{L}_2/\mathbf{L}_1 = 2$.

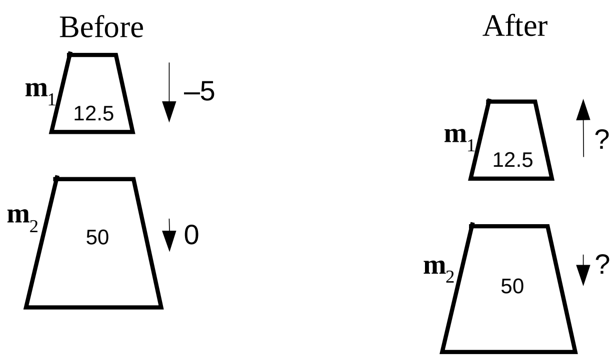
An example is shown below with $\mathbf{L}_2$ = 14 ft and $\mathbf{L}_1$ = 7 ft. Now the 400-lb weight falling $\mathbf{d}_1$ = 3 ft can transfer all its energy to the 100-lb weight, which then rises $\mathbf{d}_2$ = 12 ft.



Problems

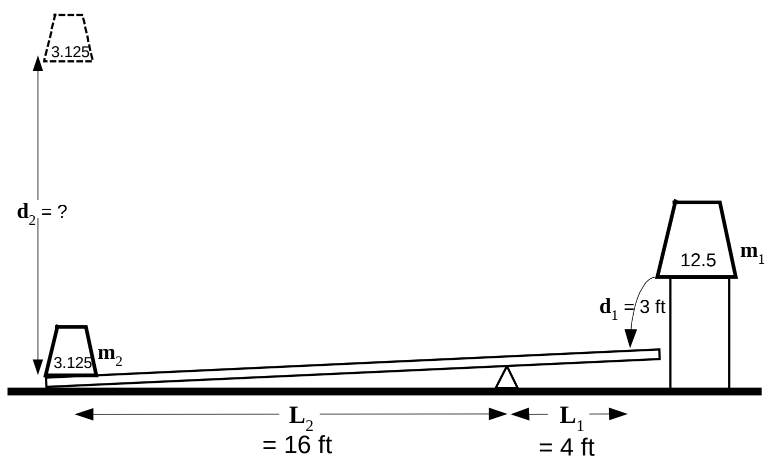
Problem 1

A mass $\mathbf{m}_1$ = 12.5 slugs with velocity –5 hits a stationary mass $\mathbf{m}_2$ = 50 slugs. What will the velocity of $\mathbf{m}_2$ be after the collision? Calculate the kinetic energy of $\mathbf{m}_1$ before the collision and the kinetic energy of $\mathbf{m}_2$ before the collision. What fraction of the kinetic energy of $\mathbf{m}_1$ has been transferred to $\mathbf{m}_2$? (See Lesson 1 for solving for the "After" velocities.)



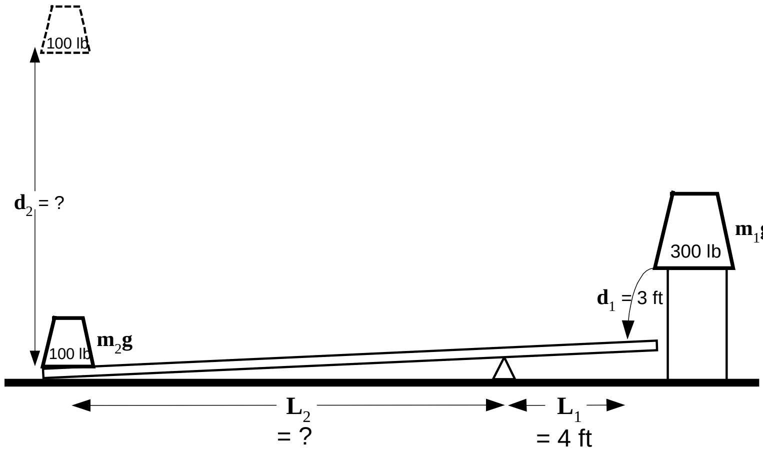
Problem 2

We saw in "First Try at Fulcrum Position" that if $\mathbf{m}_2$ = 3.125 slugs, $\mathbf{L}_2$ = 16 ft and $\mathbf{L}_1$ = 4 ft, then $\mathbf{m}_{felt}$ is 50 slugs. Use the results of Problem 1 to find the fraction of the potential energy of the falling $\mathbf{m}_1$ (12.5 slugs) that will be transferred to $\mathbf{m}_2$. How high will $\mathbf{m}_2$ go when $\mathbf{m}_1$ falls 3 ft?



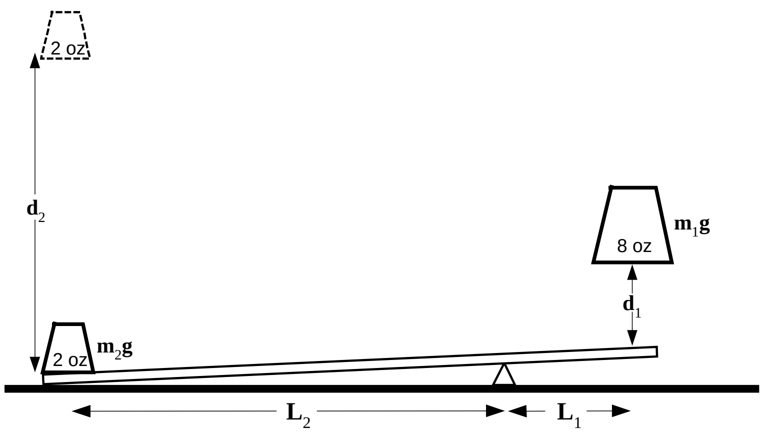
Problem 3

We reduce the weight of $\mathbf{m}_1$ to 300 lb (see figure below). Calculate the proper value for $\mathbf{L}_2$ below so that all the energy is transferred to $\mathbf{m}_2$ with weight 100 lb. (Use the method in "Second Try at Fulcrum Position.") Find the height $\mathbf{d}_2$.



Experiments

Experiment 1



Using the $\mathbf{m}_1\mathbf{g}$ = 8 oz and $\mathbf{m}_2\mathbf{g}$ = 2 oz weights provided, set $\mathbf{L}_2 = 2 \cdot \mathbf{L}_1$, and measure the ratio $\mathbf{d}_2/\mathbf{d}_1$. (Have a partner hold rulers at $\mathbf{d}_1$ and $\mathbf{d}_2$.) The ratio will be somewhat less than 4 because some of the energy from $\mathbf{m}_1$ is going into the teeter-totter's motion. Calculate what fraction of the energy from $\mathbf{m}_1$ is going to $\mathbf{m}_2$.

Experiment 2

Repeat Experiment 1, but setting $\mathbf{L}_2 = 4 \cdot \mathbf{L}_1$. Compare the measured ratio $\mathbf{d}_2/\mathbf{d}_1$ with the results of Problem 2. Calculate what fraction of the energy is going to $\mathbf{m}_2$.