# Chapter 5
# Work and Friction

We have been looking at systems without friction—that bounce and swing without losing energy. The energy may change from kinetic (motion) to potential (gravity and springs) and back again, but it stays the same—it's conserved. Friction causes things to slow down, and the energy is apparently lost. But it's actually still there in thermal energy—in the motion of vibrating molecules in the materials involved in the friction.

In Lesson 1 we were able to calculate velocities after two balls collided by using the conservation of energy and the conservation of momentum. But what if friction causes energy to not be conserved. Can we still calculate the velocities?

**Example 1**

Two balls of putty travel toward each other and stick instead of bouncing. What is the velocity of the final blob?
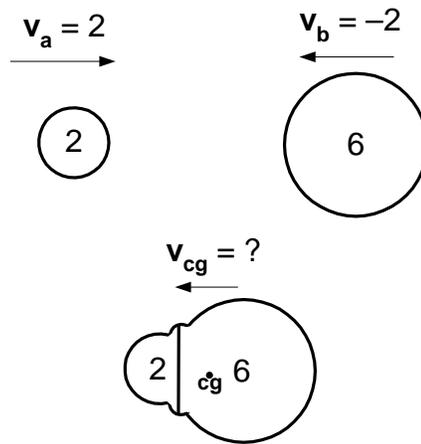


In this case kinetic energy is not conserved because the "internal" energy is lost to heat in the friction of the two balls squishing. (See Lesson 4 for what internal energy is.) But momentum must be conserved so that the energy $E_{cg}$ of the center of gravity is conserved.

Energy:     $E = \frac{1}{2}\,m\,v^2$

Momentum: $P = m\,v$

Before collision:

| $v_a$ | $v_a^2$ | $m_a$ | $E_a$ | $P_a$ |
|---|---|---|---|---|
| 2 | 4 | 2 | 4 | 4 |

| $v_b$ | $v_b^2$ | $m_b$ | $E_b$ | $P_b$ |
|---|---|---|---|---|
| −2 | 4 | 6 | 12 | −12 |

| | | $m_{total}$ | $E_{total}$ | $P_{total}$ |
|---|---|---|---|---|
| | | 8 | 16 | −8 |

$P_{total} = m_{total}\,v_{cg}$,   so   $v_{cg} = P_{total}/m_{total} = $ ==−1==.

$E_{cg} = \frac{1}{2}m_{total}v_{cg}^2 = 8$

$$\mathbf{E}_{internal} = \mathbf{E}_{total} - \mathbf{E}_{cg} = 16 - 8 = 8$$

After collision the total mass and the total momentum must remain the same:

| $\mathbf{v}_{cg}$ | $\mathbf{v}_{cg}^2$ | $\mathbf{m}_{total}$ | $\mathbf{E}_{total}$ | $\mathbf{P}_{total}$ |
|---|---|---|---|---|
| $-1$ | $1$ | $8$ | $8$ | $-8$ |

$$\mathbf{E}_{cg} = \tfrac{1}{2}\mathbf{m}_{total}\mathbf{v}_{cg}^2 = \mathbf{8}$$
$$\mathbf{E}_{internal} = \mathbf{E}_{total} - \mathbf{E}_{cg} = 8 - 8 = 0$$

So the center of gravity keeps going at a velocity of $\mathbf{v}_{cg} = -1$, but the internal energy seems to have disappeared. Actually, it's no longer visible as kinetic energy, but it's now in the form of thermal energy (the putty is warmer).

# Work

Work is defined as a force through a distance:

$$\mathbf{W} = \mathbf{F}\cdot\mathbf{d}$$

It's called work because it's useful—lifting water, driving a nail, pushing a saw. Work is an indication that energy is being transformed from one form to another. The chemical energy in our breakfast cereal (with the help of our muscles) lifts a can of beans to give it potential energy. When we release the can, gravity does work on the can, accelerating it to give it kinetic energy. If the can hits a nail, it does work forcing the nail into the wood, producing thermal energy (the nail gets hot). In each case the work is equal to the energy producing the work and equal to the energy it produces.

**Example 2**

The weight of a 1-lb can is the force $\mathbf{F}$ needed to oppose gravity:

$$\mathbf{F} = \mathbf{m}\cdot\mathbf{g} = 1 \text{ lb},$$

where $\mathbf{m}$ is the mass and $\mathbf{g}$ is the acceleration due to gravity. Suppose you lift the can $\mathbf{d} = 4$ ft. Your work is

$$\mathbf{W} = \mathbf{F}\cdot\mathbf{d} = \mathbf{m}\cdot\mathbf{g}\cdot\mathbf{d} = (1 \text{ lb})\cdot(4 \text{ ft}) = 1 \text{ ft-lb}.$$

But $\mathbf{m}\cdot\mathbf{g}\cdot\mathbf{d}$ is the potential energy of the lifted can (see Lesson 2):

$$\mathbf{E}_P = \mathbf{m}\cdot\mathbf{g}\cdot\mathbf{d} = 4 \text{ ft-lb}.$$

So the work equals the energy it produces. As the can falls, gravity pulls on it with the same force (1 pound) through the same distance (4 feet), doing 4-ft-lb of work on it and giving it an equal amount of kinetic energy:

$$\mathbf{E}_K = \tfrac{1}{2}\mathbf{m}\cdot\mathbf{v}^2 = 4 \text{ ft-lb}$$

Using the methods in Lesson 2 we can find $\mathbf{v} = 16$ ft/sec. That is, a mass weighing 1 lb with a velocity of 16 ft/sec has *energy* of 4 ft-lb, and it can do useful *work* equaling 4 ft-lb. For instance, it could drive a nail into a wooden board. (It's a steel can so the nail won't puncture it.)
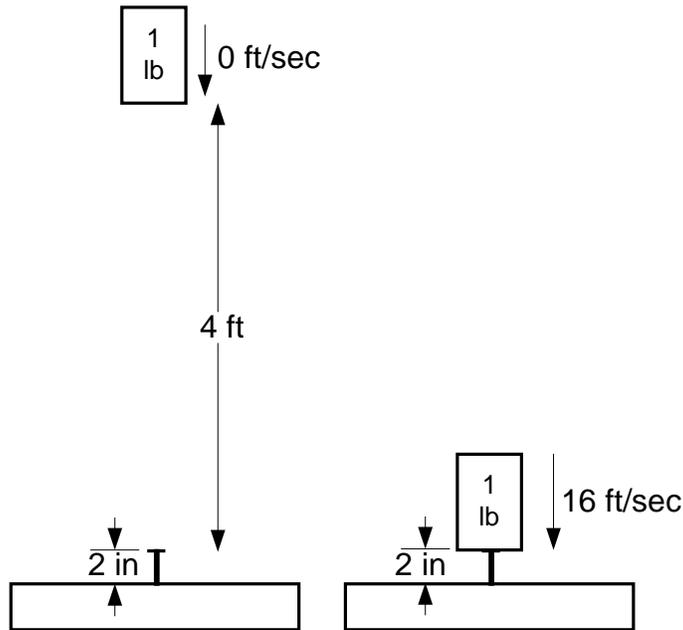
Let's say the nail is sticking 2 inch out of the wood before the can hits it, and the can drives it a distance $d_n = 1$ inch as it comes to rest (has used all its kinetic energy).
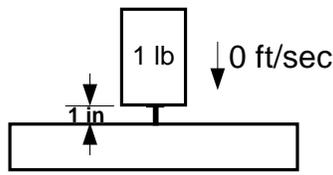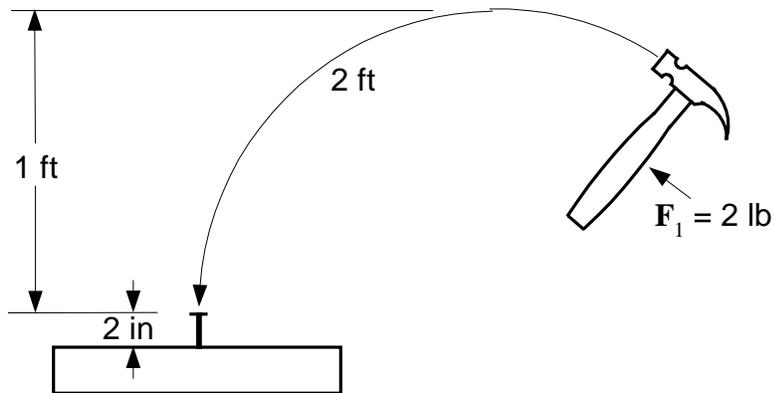


What was the force $F_n$ needed to push the nail?  Well, the distance the nail was pushed was $d_n = 1$ in $= \frac{1}{12}$ ft, and the work $W_n$ done on the nail is equal to the 4 ft-lb of kinetic energy:

$$W_n = F_n \cdot d_n = F_n \cdot (\frac{1}{12} \text{ ft}) = 4 \text{ ft-lb}$$

so $$F_n = 4/(\frac{1}{12}) = \text{48 lb}$$

**Example 3**



Now, 48 lb is probably more force than you could apply by simply *pressing* the nail with a hammer.  But you could *pound* with the hammer, accelerating it by exerting $F_1 = 2$ lb of force through an arc distance $d_1 = 2$ ft.  Your work is

$$W_1 = F_1 \cdot d_1 = 4 \text{ ft-lb}.$$

At the same time gravity is pulling downward on the 1-lb hammer with $F_2 = 1$ lb through a height of $d_2 = 1$ ft (the distance in the direction of the force of gravity). Gravity's work is

$$W_2 = F_2{\cdot}d_2 = 1 \text{ ft-lb.}$$

Then the total work done is

$$W = W_1 + W_2 = 4 + 1 = 5 \text{ ft-lb.}$$

The total kinetic energy the hammer has when it strikes the nail is equal to the work done on it:

$$E_K = W = 5 \text{ ft-lb.}$$

If it takes $F_n = 48$ lb of force to move the nail, how far $d_n$ will the hammer with kinetic energy $E_K = 5$ ft-lb drive the nail? The work $W_n$ it does on the nail equals $E_K$:

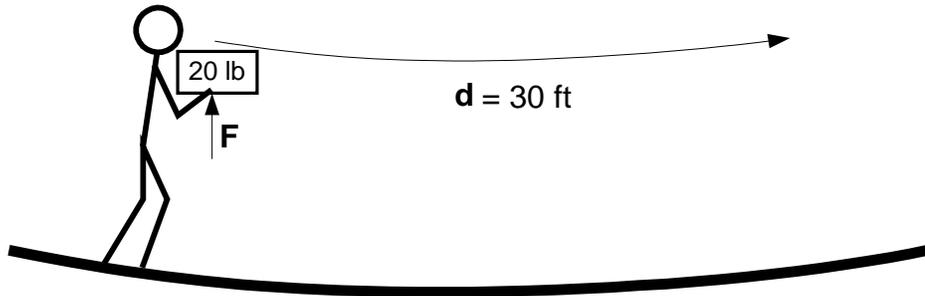$$W_n = F_n{\cdot}d_n = (48 \text{ lb}){\cdot}d_n = 5 \text{ ft-lb}$$

so
$$d_n = {}^5\!/_{48} \text{ ft} = 1{}^1\!/_4 \text{ inches.}$$

You would need a second blow of the hammer to drive it the remaining ¾ inch.

## Example 4

You carry a weight of 20 lb a distance of 30 ft. How much work have you done?



Since the force is not in the same direction as the motion (it's at 90° to the direction of motion), you haven't done any work. It seems to you that holding a weight is hard work, but a cart can support the weight easily, and it can move it with no work.
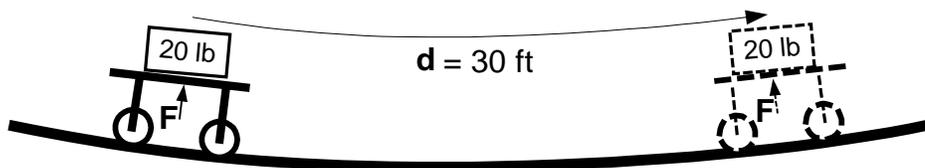


The cart is on a slightly curved floor. It starts at rest with the 20-lb weight on it. The slope makes it start to move to the right. After it's gone 30 ft, the up-grade has slowed the cart so it stops (at the same height it started). So the cart started with zero kinetic energy and ended with zero kinetic energy. It has the same potential energy at the end that it started with. So no work has been done. Again, the motion was not in the direction of the 20-lb force.

# Friction

You pull a sliding 4-lb weight horizontally through a distance $d = 10$ feet. Are you doing work now? Yes, the force $F$ is in the direction of motion. But what is $F$? Is it 4 lb? Probably not. The force you need is that due to friction, not the force of 4 lb needed to lift the weight.

**d** = 10 ft
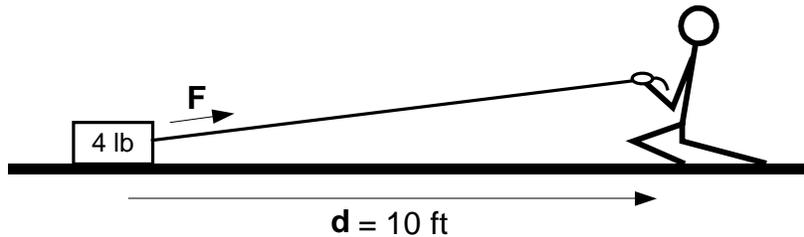
The force of friction depends on the weight of the object, on the material on the bottom of the object—metal or wood—and on the surface it's being pulled over—grass or ice. There's no way to calculate how much the force of friction will be; it has to be measured. But experiments have shown that the force **F** of friction is proportional to the weight **m·g** of the object. It takes twice as much force to pull an 8-lb weight over grass as it does a 4-lb weight. So we can write

$$\mathbf{F} = \mathbf{k} \cdot \mathbf{mg}$$

where **k** is the coefficient of friction and **mg** is the weight. The coefficient may be high for wood on carpet, and it may be low for metal on wood.

## Example 5

Suppose you pull a 4-lb frying pan a distance **d** = 10 feet on a carpet surface, and the force **F** required is 1.5 lb. What is the friction coefficient **k**, and what is the work **W** done?
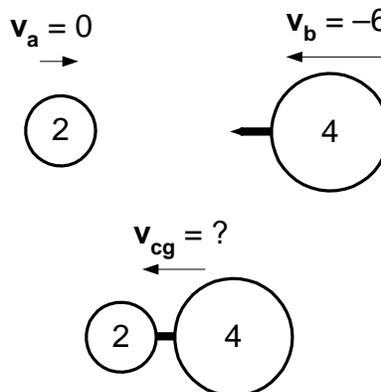
$$\mathbf{k} = \mathbf{F}/\mathbf{mg} = (1.5 \text{ lb})/(4 \text{ lb}) = 0.375$$

$$\mathbf{W} = \mathbf{F} \cdot \mathbf{d} = (1.5 \text{ lb}) \cdot (10 \text{ ft}) = 15 \text{ ft-lb}$$

The coefficient **k** is usually less than 1. It can be very small if you're pulling smooth metal over ice.
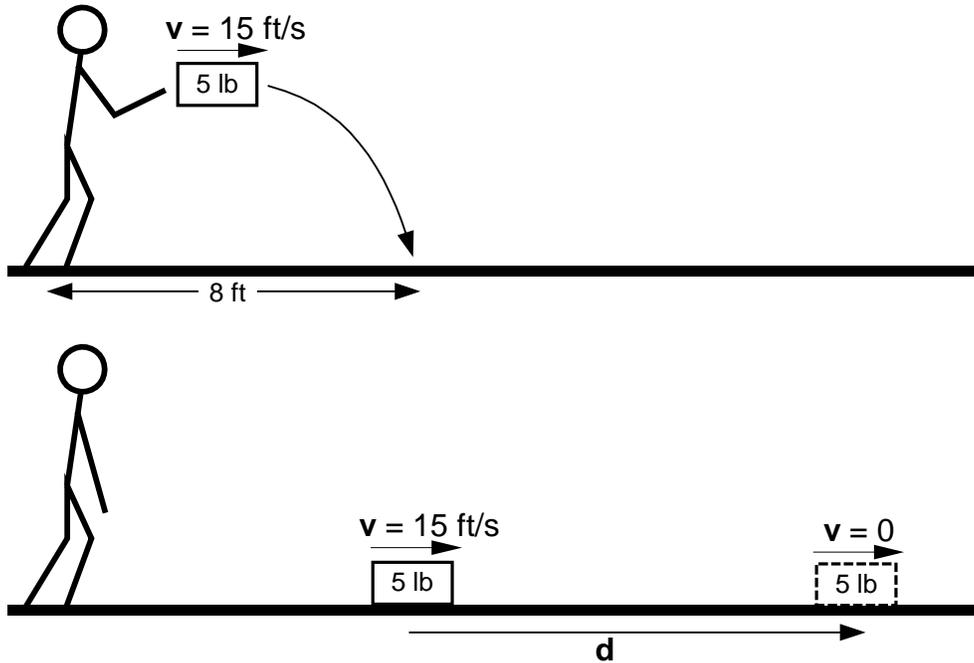
# Problems

## Problem 1



A 4-lb ball with a velocity of –6 is going toward a 2-lb wood ball at rest. A nail fastened to the 4-lb ball sticks into the 2-lb ball when they collide. What is the final velocity **v**$_{cg}$ of the system after the balls are joined? How much energy is converted to heat from the friction of the nail?

**Problem 2**



You throw a 5-lb block with a velocity $v = 15$ ft/s. It hits the ground 8 ft from you and begins sliding at 15 ft/s. It slides a distance **d** before friction stops it. The coefficient of friction was measured to be **k** = 0.2

The block's weight is **m·g** = 5 lb, and the acceleration of gravity is **g** = 32 ft/s/s. What is the block's mass **m** (in slugs)? What was the block's initial kinetic energy? How much work did the block do before it came to a stop? (Remember, the work an object does equals the energy it initially had.) What was the force **F** of friction? How far **d** did the block slide?

**Problem 3**
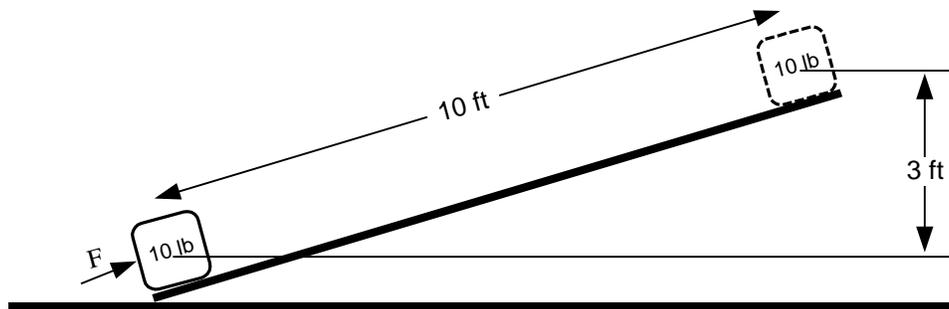


You slide a 10-lb block of ice a distance $d_1 = 10$ feet up a ramp, raising the block's center of gravity by $d_2 = 3$ ft. There is effectively no friction; **k** = 0. How much potential energy did the block gain? How much work did you do? (Remember, the work done on an object equals the energy it gained.) What was the force **F** you had to push the block with?

# Experiments

**Experiment 1**

Paste a piece of paper on the home-made weight scale so you can draw lines and numbers on it. Let the hook hang with no weight, and put "0" opposite the end of the spring. Now hang a 1-to-2 lb weight (as measure by a kitchen scale) on the hook, and write the weight opposite the end of

the spring.  Fill in other numbers based on the two you have.  Try other weights on your scale, and see if it is calibrated properly.

0 —

0 —

1 —

1 lb
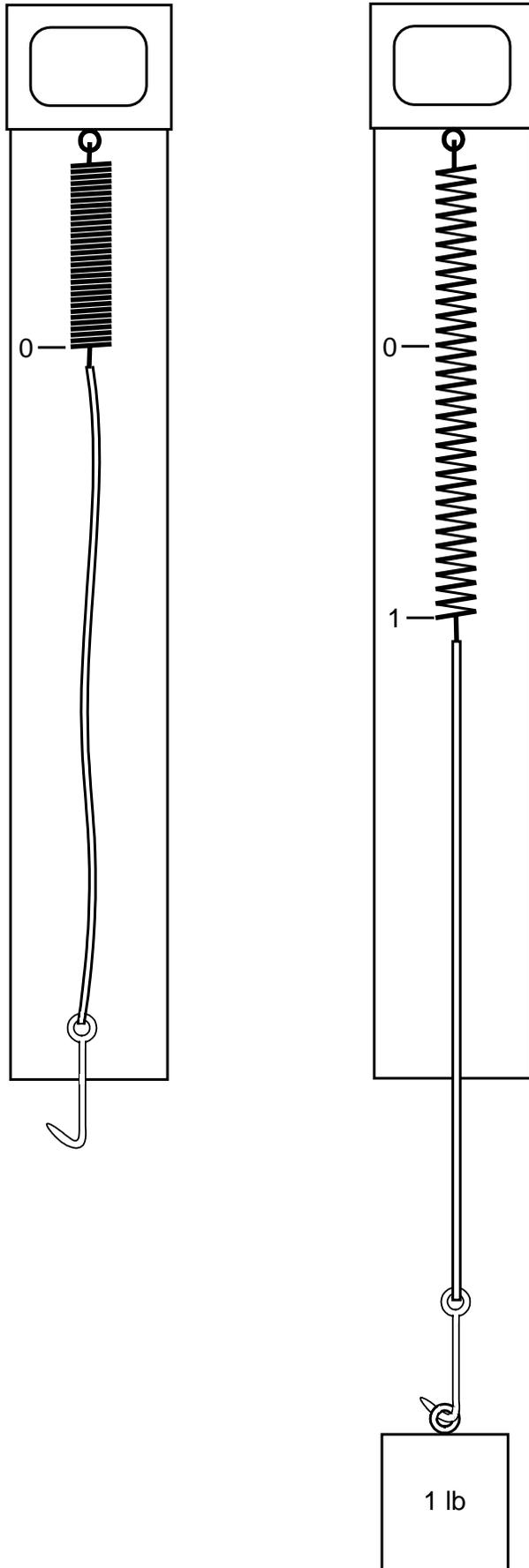
**Experiment 2**

Measure the weight of a frying pan. (Use a kitchen scale if it's too heavy for the scale you made.) Pull the frying pan across the carpet with your scale and measure the force required. Notice that the initial "stiction" is greater than the friction as you're pulling rapidly. What is the coefficient of friction? Find the coefficient of friction for other surfaces. Try pulling other weights. What's the largest coefficient of friction you could find?

**Experiment 3**

Measure the weight of a hammer and drop it from a height of about a foot onto a small nail stuck in a piece of wood. What was the initial potential energy of the hammer? How far did the hammer drive the nail? What must the force have been to drive the nail? (See Example 2.)

Repeat the experiment, but this time drive the nail by swinging the hammer with good force through an arc (see Example 3). How long was the arc? Measure how far you drove the nail this time. What force must you have applied to the hammer as it swung? (Remember that gravity helped.)