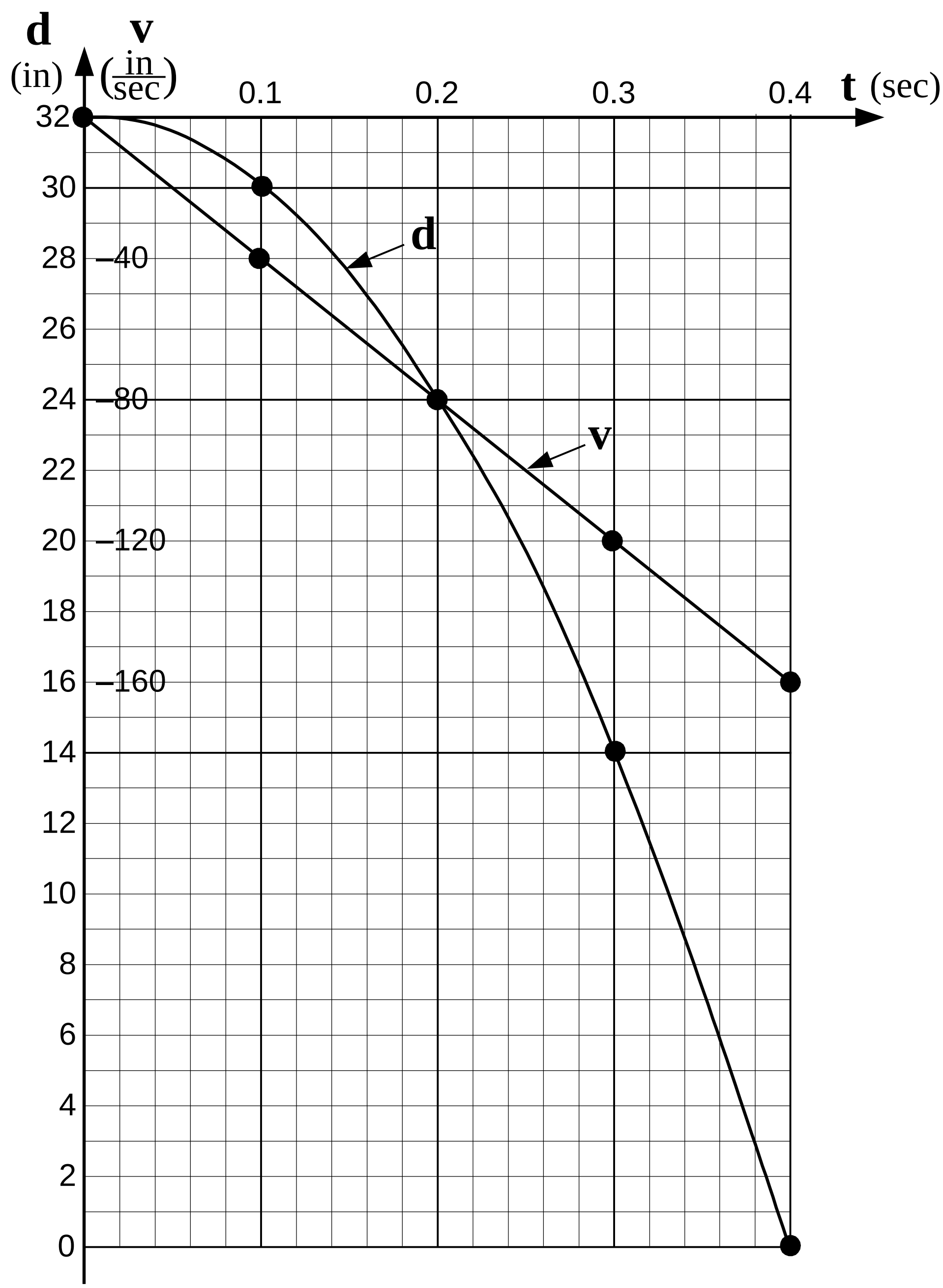
Chapter 3
Potential Energy

Example 1

Let's look again at Example 3 in Lesson 2, where a ball was falling under the acceleration of gravity. We plotted the distance **d** it fell as a function of time **t**, and we plotted it's velocity **v** as of function of time. The plot below combines the two plots on one grid. Note we have chosen to call the height **d** = 0 at **t** = 0.4 sec because the ball can't go any lower (it hits the ground or something). The ball is dropped from a height **d**= 32 inches.



At t = 0 the ball has just been dropped, and it has no velocity. But as time goes on it falls faster and faster, gaining kinetic energy

$$\mathbf{E_K = \tfrac{1}{2}\, m \cdot v^2}$$

Let's calculate $\mathbf{E}_K$ as $\mathbf{v}$ increases with time $\mathbf{t}$.

t	**v**	$\mathbf{v}^2$	$\mathbf{E}_K$
0.0	0	0	0
0.1	–40	1600	800·**m**
0.2	–80	6400	3200·**m**
0.3	–120	14400	7200·**m**
0.4	–160	25600	12800·**m**

So the kinetic energy increases with time. But conservation of energy says the total energy must be conserved. So there must be some form of energy decreasing at the same time.

When the ball is a distance **d** above the ground it has the potential to fall and gain kinetic energy. As the distance decreases, that potential is less. We define the *potential energy* as

$$\mathbf{E_P = -a \cdot m \cdot d} = 400 \cdot \mathbf{m \cdot d}$$

where **a** = –(400 in/sec)/sec is the acceleration due to gravity. In the table below we calculate $\mathbf{E}_P$ as **d** decreases with time **t**. We also list $\mathbf{E}_K$.

t	**d**	$\mathbf{E}_P$	$\mathbf{E}_K$	$\mathbf{E}_P + \mathbf{E}_K$
0.0	32	12800·**m**	0	12800·**m**
0.1	30	12000·**m**	800·**m**	12800·**m**
0.2	24	9600·**m**	3200·**m**	12800·**m**
0.3	14	5600·**m**	7200·**m**	12800·**m**
0.4	0	0	12800·**m**	12800·**m**

We see that the total energy $\mathbf{E}_P + \mathbf{E}_K$ is a constant 12800·**m**, and energy is conserved. In particular, the initial potential energy (at **t**= 0) equals the final kinetic energy (at **d** = 0). This fact lets us calculate the final velocity $\mathbf{v}_{final}$ from the initial height $\mathbf{d}_{initial}$.

The initial potential energy is

$$\mathbf{E_P = -a \cdot m \cdot d}_{initial}$$

And the final kinetic energy is

$$\mathbf{E_K} = \tfrac{1}{2}\, \mathbf{m \cdot v}_{final}^2$$

But these two energies are equal. Therefore

$$\tfrac{1}{2}\, \mathbf{m \cdot v}_{final}^2 = \mathbf{-a \cdot m \cdot d}_{initial}$$

$$\mathbf{v}_{final}^2 = -2 \cdot \mathbf{a} \cdot \mathbf{d}_{initial} = 800 \cdot \mathbf{d}_{initial}$$

For our example, $\mathbf{d}_{initial}$ = 32 inches, so $\mathbf{v}_{final}^2$ = 25600 (in/sec)^2, and $\mathbf{v}_{final}$ = –160 in/sec. (Try squaring –160 to see this gives $\mathbf{v}_{final}^2$ = 25600.)

Now we don't have to plot the whole graph just to find the final velocity. Suppose we drop a ball from an initial height $\mathbf{d}_{initial}$ = 50 inches. What is the final velocity?

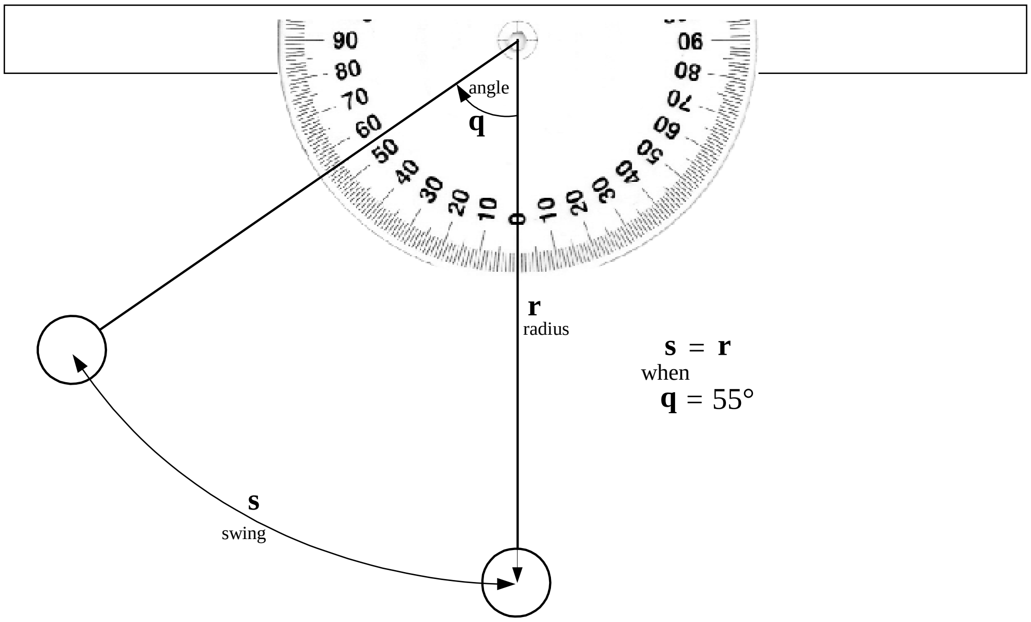
$$\mathbf{v}_{final}^2 = = 800 \cdot \mathbf{d}_{initial} = 40{,}000 \text{ (in/sec)}^2$$

and $\mathbf{v}_{final}$ = 200 in/sec. Can we check this experimentally? We can measure the initial height $\mathbf{d}_{initial}$ = 50 inches with a tape ruler, but how do we measure the final velocity to be $\mathbf{v}_{final}$ = 200 in/sec? It would be difficult.

Example 2

In the experiments in Lesson 1 we related the velocity of a swinging ball when it reached the center position to its initial distance from the center poison. So here is a way to measure velocity by measuring a distance. We just need to be quantitative about it.

Since a ball on a string (a *pendulum*) travels in a curved path, it's more correct to measure the distance of its swing along a circle rather than a straight line (as we did in Lesson 1). But it's hard to bend a tape ruler in a nice circle, so we'll use a protractor instead, as in the picture below. We swing the pendulum a distance **s** (along a curved path) so that **s** is

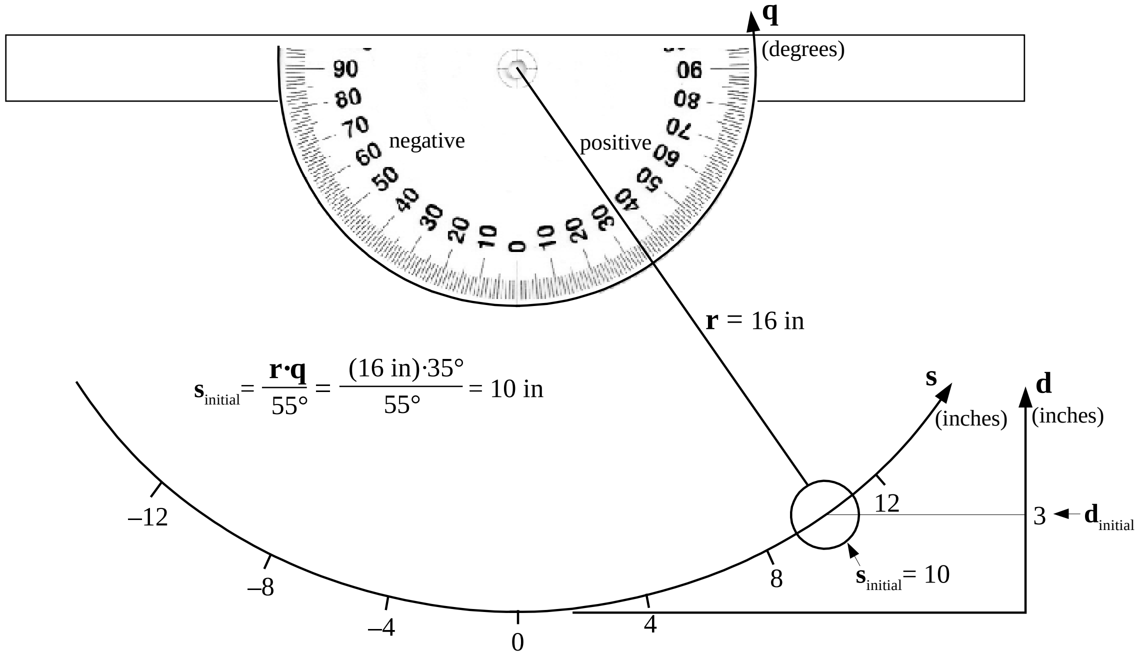


the same length as the radius **r** (always measuring to the center of the ball). The corresponding angle **q** on the protractor is 55°. Since **s** is proportional to **q**, we can say that for any swing **s**,

$$\mathbf{s} = \mathbf{r} \cdot \mathbf{q}/55°$$

For example, if we know that **r** = 16 inches and **q** = 35°, then **s** = (16 in)·35°/55° = 10 inches.

Now we're ready to check that the potential energy lost equals the kinetic energy gained. We pull the pendulum back to an initial **q** = 35° corresponding to an initial pendulum position $\mathbf{s}_{\text{initial}}$ = 10 inches. The lowest the ball can go is when **q** = 0° (at the center of the



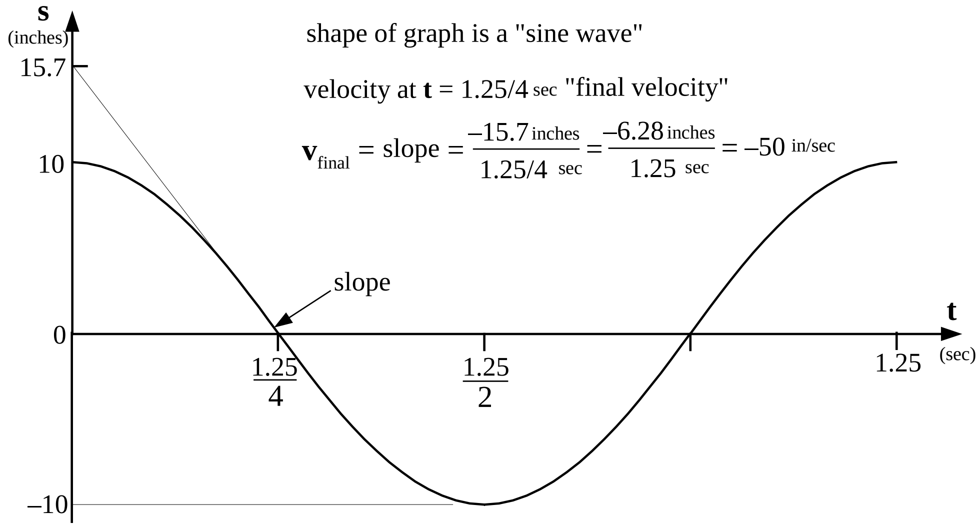
swing), so we measure the height **d** as the distance above the bottom of the swing. This will give us its potential energy. We measure the initial height to be $\mathbf{d}_{\text{initial}}$ = 3 inches, corresponding to a potential energy of $\mathbf{E}_P = -\mathbf{a}\cdot\mathbf{m}\cdot\mathbf{d}_{\text{initial}}$ = (400 in/sec/sec)·**m**·(3 in). When all of this is converted into kinetic energy at the bottom of the swing, the final velocity will be

$$\mathbf{v}_{\text{final}}^2 = 800\cdot\mathbf{d}_{\text{initial}} = 2400\ (\text{in/sec})^2.$$

What number times itself gives 2400? $\mathbf{v}_{\text{final}}$ = –50 in/sec is too large, but $\mathbf{v}_{\text{final}}$ = –49 in/sec gives $\mathbf{v}_{\text{final}}^2 = 2401\ (\text{in/sec})^2$, which is pretty close.

This value of $\mathbf{v}_{\text{final}}$ is just a calculation based on trust that the final kinetic energy equals the initial potential energy. But we want to actually measure $\mathbf{v}_{\text{final}}$ to see if our trust is right. If we plot the swing **s** as a function of time **t** we can find the velocity from the slope of the plot.

Things that vibrate—like a pendulum or a piano string or a chime—move in a sine wave like that shown below. The peak value or *amplitude* of the sine wave is $\mathbf{s}_{\text{initial}}$ = 10, the distance that we initially pull the pendulum. Immediately after we release it, the velocity is zero, as we can see from the zero slope (horizontal) at **t** = 0. As **s** passes through **s** = 0 (the center position when the ball is at the lowest point) the slope is steepest (the velocity is greatest). If we draw a straight line to extend that slope, it touches the **s** axis 57% higher than $\mathbf{s}_{\text{initial}}$, that is, at 15.7 inches.

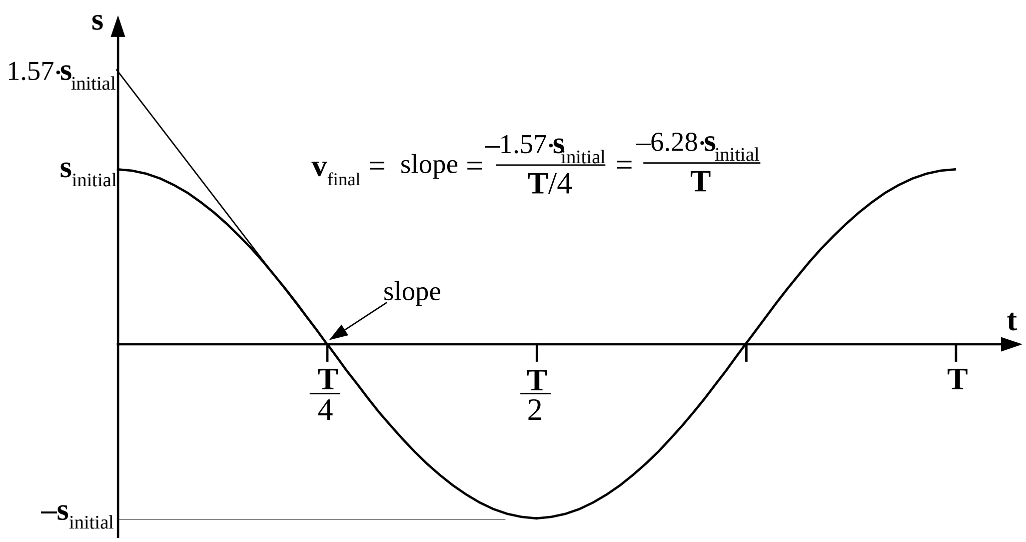


If we can find the time when the plot crosses **s** = 0, we can calculate the slope. That time is ¼ of a complete *cycle* (the pendulum returning to where it started). We can use a watch to measure how long one cycle takes. (Actually, I measured that 8 cycles took 10 seconds, so one cycle takes (10 sec)/8 = 1.25 sec.) Then our triangle at the left has a width of 1.25/4 and a height of 15.7, and the slope at **t** = 1.25/4 is

$$\mathbf{v}_{final} = \frac{-15.7\text{ in}}{1.25/4\text{ sec}} = -50\text{ in/sec}$$

which is close to the value $\mathbf{v}_{final}$ = 49 in/sec that we calculated from the initial height $\mathbf{d}_{initial}$.

The plot below generalizes the results we saw above. The starting position of the pendulum is $\mathbf{s}_{initial}$—how far it is from the center or “rest” position. The length of time for



one cycle (the *period*) is **T**. The “final” velocity (when the pendulum crosses the center position **s** = 0) is given by

$$\mathbf{v}_{final} = \frac{-6.28\cdot\mathbf{s}_{initial}}{\mathbf{T}}$$

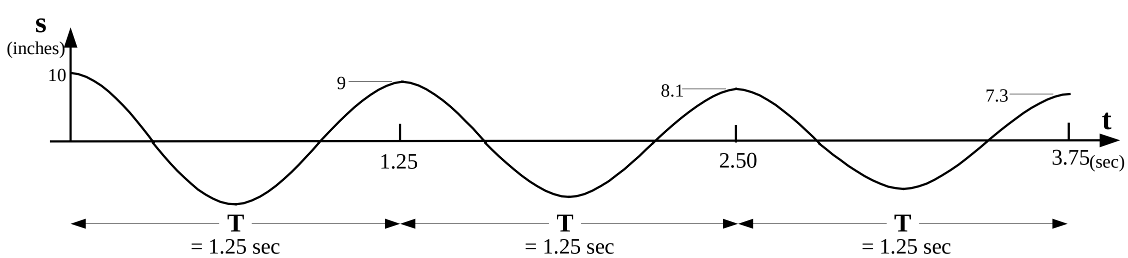
We can measure **T** more accurately by measuring the time for 8 cycles and dividing by eight (or some other number of cycles). But does the period stay the same as the pendulum loses energy from air friction? We saw that the period of a bouncing ball got shorter as the ball lost energy due to internal friction when it compresses. But a bouncing ball is not a vibration with a sine wave motion.

It happens that the period **T** of a vibration stays almost the same no matter how far the vibration swings. The frequency of a piano string doesn't depend on how hard you hit it. The frequency **f** is the number of cycles per second, and the period **T** is the number of seconds per cycle. So we find the frequency from the period by

$$\mathbf{f} = 1/\mathbf{T}.$$

A period of 1.25 seconds is a frequency of **f** = 1/1.25 = 0.8 cycles per second, or 0.8 *Hertz*. (Hertz is a short way of saying "cycles per second.") The unit Hertz is abbreviated Hz. A period of 0.0038 seconds is a frequency of 263 Hz (middle C on the piano). Generally if the period is less than one second (**T** < 1 sec), then we talk about the frequency **f**. If the period is longer than one second (**T** > 1 sec), then we talk about the period of vibration **T**.

Since the frequency of a vibration doesn't change as the amplitude $\mathbf{s}_{\text{initial}}$ gets less, then the period **T** doesn't change either. The plot below illustrates this. The amplitude of the first cycle of the pendulum is 10 inches, the amplitude of the second cycle is 9 inches, and the amplitude of each succeeding cycle is 10% less than the previous. But the period remains the same at **T**= 1.25 seconds. This lets us find **T** by measuring the time for a number of cycles and dividing by that number.



One question remains about converting potential energy $\mathbf{E}_P$ into kinetic energy $\mathbf{E}_K$. For a falling ball, gravity pulls straight down and causes a velocity **v** that is straight down, and we got the relationships

$$\mathbf{E}_P = -\mathbf{a}\cdot\mathbf{m}\cdot\mathbf{d}_{\text{initial}}$$

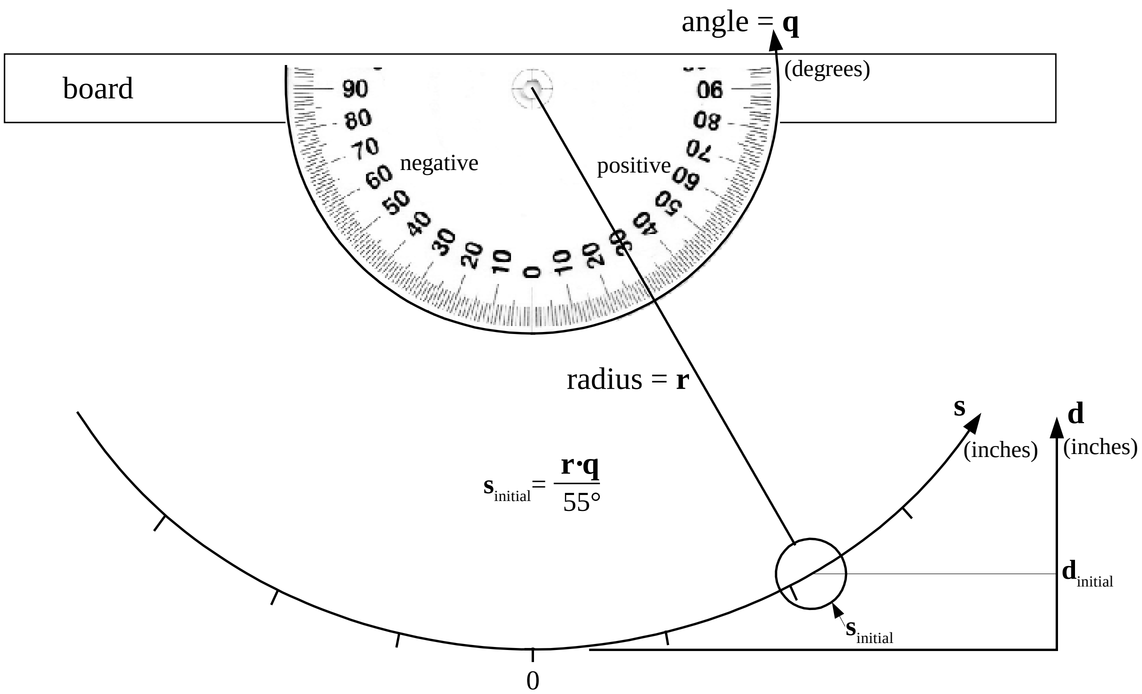
$$\mathbf{E}_K = \tfrac{1}{2}\,\mathbf{m}\cdot\mathbf{v}_{\text{final}}^2$$

And since $\mathbf{E}_P = \mathbf{E}_K$, we can find the velocity from the initial height: $\mathbf{v}_{\text{final}}^2 = -2\mathbf{a}\cdot\mathbf{m}\cdot\mathbf{d}_{\text{initial}}$, where **a** = –400 in/sec/sec. But a pendulum bends $\mathbf{v}_{\text{final}}$ around so it's going sideways, not straight down. Do the relationships still hold? This is the advantage of using conservation of energy to solve problems. Direction doesn't matter; only the total energy matters. Since the pendulum doesn't lose any energy to heat as it changes the direction of the ball (if we neglect air friction), then all of the potential energy (the energy of height) becomes kinetic energy (the energy of motion). You can prove this with an experiment.

Experiments

Experiment 1

Construct the pendulum shown below by gluing the protractor (see last page) to the board and pounding a nail into the center of the circle. Tie a string to the nail and pass the other end through the hole in the ball. Tie a large knot in that end so the ball doesn't slide off. Make the string as long as you can so it doesn't hit the supporting chairs when you pull the ball to an angle of 45°. This will make distances and times longer so they are easier to measure.



Pull the ball to some angle **q** and release it. Time a number of cycles and find the period **T**. Try this for different of values of **q** and see if **T** remains the same.

Experiment 2

Let the ball hang (**q** = 0) and measure the distance of the center of the ball from the floor. Pull the ball to $\mathbf{q}_{initial}$ = 30° and measure the new distance of the center of the ball from the floor. The difference between the two measurements is the initial height $\mathbf{d}_{initial.}$

Release the ball, and use $\mathbf{d}_{initial}$ to calculate the final velocity $\mathbf{v}_{final}$ (as the pendulum reaches **d** = 0 at the bottom of the swing).

Calculate the initial position $\mathbf{s}_{initial}$ when $\mathbf{q}_{initial}$ = 30°. Use this $\mathbf{s}_{initial}$ together with your measured **T** to find $\mathbf{v}_{final}$. Does this values agree with the value you calculated from $\mathbf{d}_{initial}$?

Experiment 3

Repeat Experiment 2 for $\mathbf{q}_{initial}$ = 45°. Do the two calculated values for $\mathbf{v}_{final}$ agree?

Problems

Problem 1

A ball is dropped from a height $\mathbf{d}_{initial}$ = 50 inches. The table below gives its velocity as time increases. Fill in the corresponding values of $\mathbf{v}^2$ and of $\mathbf{E}_K$ in terms of the mass **m**.

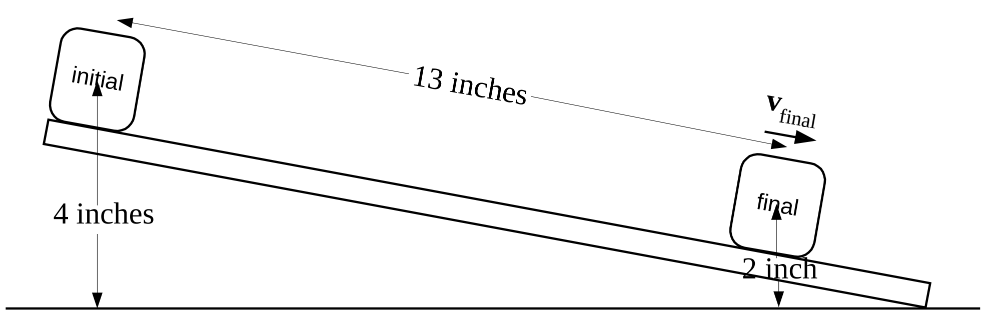
t	v	v^2	E_K
0.0	0		
0.1	–40		
0.2	–80		
0.3	–120		
0.4	–160		
0.5	–200		

The table below gives the height **d** as time increases. Fill in the corresponding vales of $\mathbf{E}_P$ in terms of the mass **m**. Add the corresponding values of $\mathbf{E}_K$, and show the total energy $\mathbf{E}_P + \mathbf{E}_K$ stays the same.

t	d	E_P	$E_P + E_K$
0.0	50		
0.1	48		
0.2	42		
0.3	32		
0.4	18		
0.5	0		

Problem 2

An ice cube is sliding down a ramp with no friction. It is released when its center is 4 inches above the table. After it slides 13 inches, its center is 2 inches above the table.



Calculate its velocity $\mathbf{v}_{final}$ at that point.

Note: The answer would be different if we used a rolling ball rather than a sliding ice cube. This is because a ball gains some of its kinetic energy in rotation, and its linear velocity would be less that the ice cube's. We'll look at this in a later lesson.