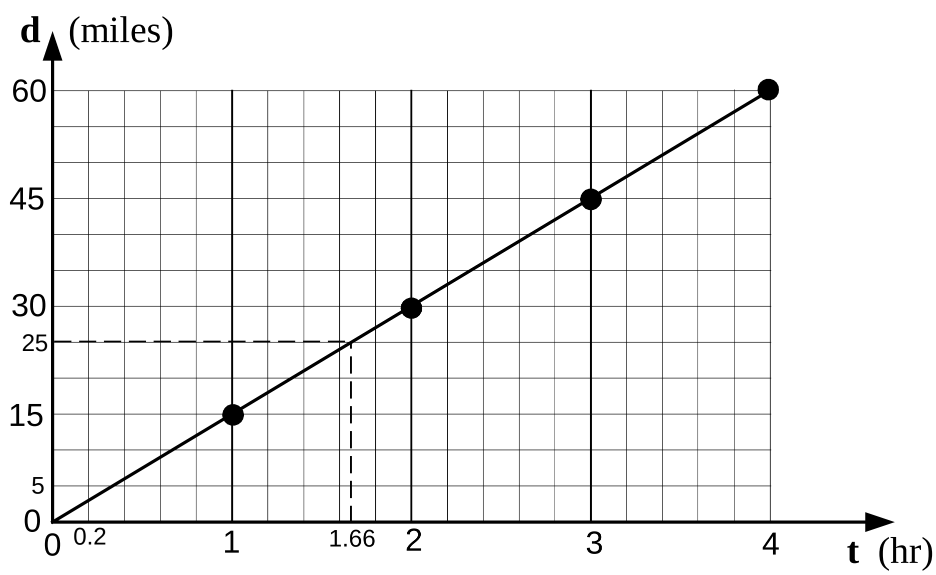
Chapter 2
Force and Acceleration

Example 1

Your dad is driving at 15 miles per hour (mph). So after one hour he's gone 15 miles, and after 2 hours he's gone 30 miles. Let **v** = 15 mph be the velocity he's going, **t** be the time he's been driving, and **d** be the distance he's gone. Then the relation is

$$\mathbf{d = v \cdot t}$$

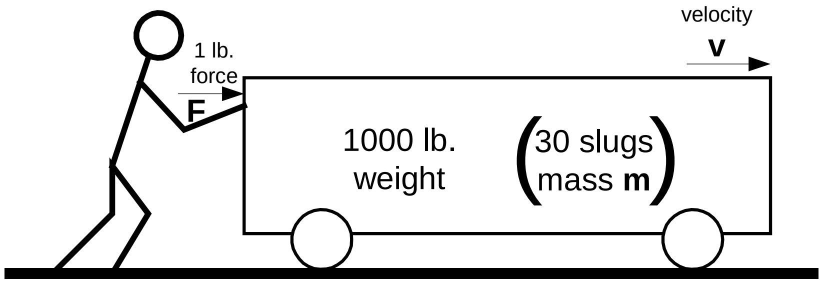
So for **t** = 2 hr, then **d** = (15 mph)·(2 hr) = 30 miles. Let's *plot* the relationship:



We choose to let horizontal *grid* lines be 5 mph apart, and let the vertical grid lines be 0.2 hr (1/5 of an hour) apart. The vertical line with an arrow is called the **d** *axis*, and the horizontal line with an arrow is called the **t** axis. Then we put a few numbers on the d axis at 15, 30, 45, and 60 mph, and we place the numbers 1, 2, 3, and 4 on the t axis. Notice each axis is also labeled with its corresponding *unit*: miles for the distance **d**, and hr for the time **t**. The dots are *points* we put on the grid to show the values **d** = 15 miles for **t** = 1 hr, **d** = 30 miles for **t** = 2 hr, **d** = 45 miles for **t** = 3 hr, and **d** = 60 miles for **t** = 4 hr. We connect the dots with a smooth line; this is the *plot* of **d** as a *function* of **t**. It lets us see values of **d** for values of **t** between the dots. For example, for **t** = 1.66 we get **d** = 25.

Notice that the plot increases by 30 as the time increases by 2. The *slope* of the line is The vertical increase divided by the horizontal increase: slope = (30 miles) / (2 hr) = 15 miles/hr, of 15 mph. Since the plot is a strait line, it doesn't matter which increase in time we take. For **t** increasing by 3, the plot rises by 45, and slope = 45/3 = 15 mph again. So the slope of the plot of distance **d** and time **t** is the velocity **v**.

Example 2



You push a railroad car that weighs 1000 ponds (lb.) with 1 lb. of *force*. The car slowly begins to move, and as you continue to push, it moves faster and faster. You find that after 3 minutes it's going 4 miles per hour (mph), and after 6 minutes it's going 8 mph. So it's gaining speed (velocity) at a rate of 4 mph per 3 minutes. We can express this as

The rate at which the velocity is increasing is called *acceleration* with the symbol **a**. In this example

$$\mathbf{a} = \frac{\text{4 mph}}{\text{3 minutes}} = \text{4/3 mph/min} = \text{1.33 mph/min}$$

Let **t** be the symbol for time. Then we find the velocity **v** by multiplying:

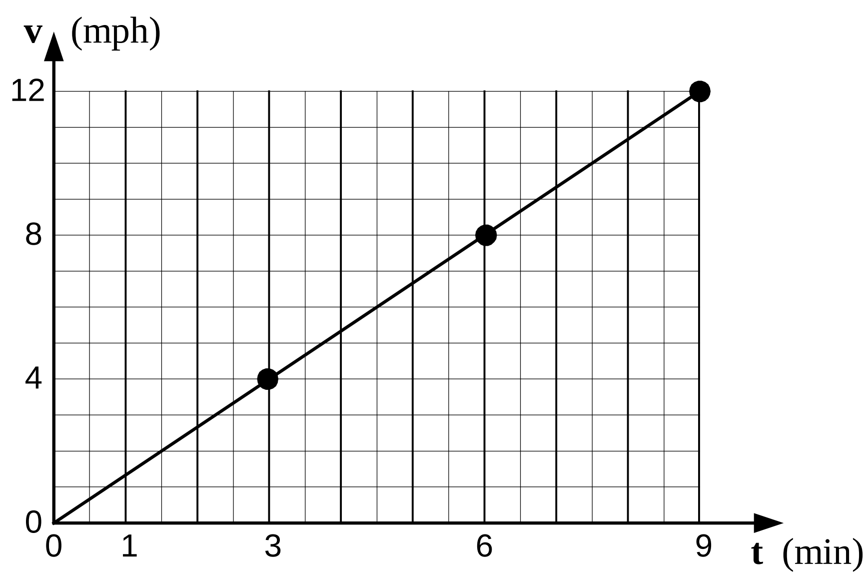
$$\mathbf{v} = \mathbf{a} \cdot \mathbf{t}$$

For **t** = 3 min, then **v** = (4/3)·3 = 4 mph, as we found before.

If you push harder, the acceleration is greater. The relation is

$$\mathbf{a} = \mathbf{F}/\mathbf{m} \quad \text{or} \quad \mathbf{F} = \mathbf{m} \cdot \mathbf{a},$$

where **m** is the mass, which is related to the weight (see Problem 5). We say the acceleration **a** is *proportional* to the force **F**. Let's plot **v** as you push for a longer and longer time **t**.



This is the *plot* of **v** as a *function* of **t**. It connects the dots we plotted for certain calculated values of **v**: **v** = 4 mph for **t** = 3 min, **v** = 8 mph for **t** = 6 min, and **v** = 12 mph for **t** = 9 min. The vertical axis is the **v** *axis*, and the horizontal axis is the **t** axis. The *units* are mph for the velocity **v**, and min for the time **t**. Notice we can see the acceleration from the plot; it's the slope 4/3 mph/min.

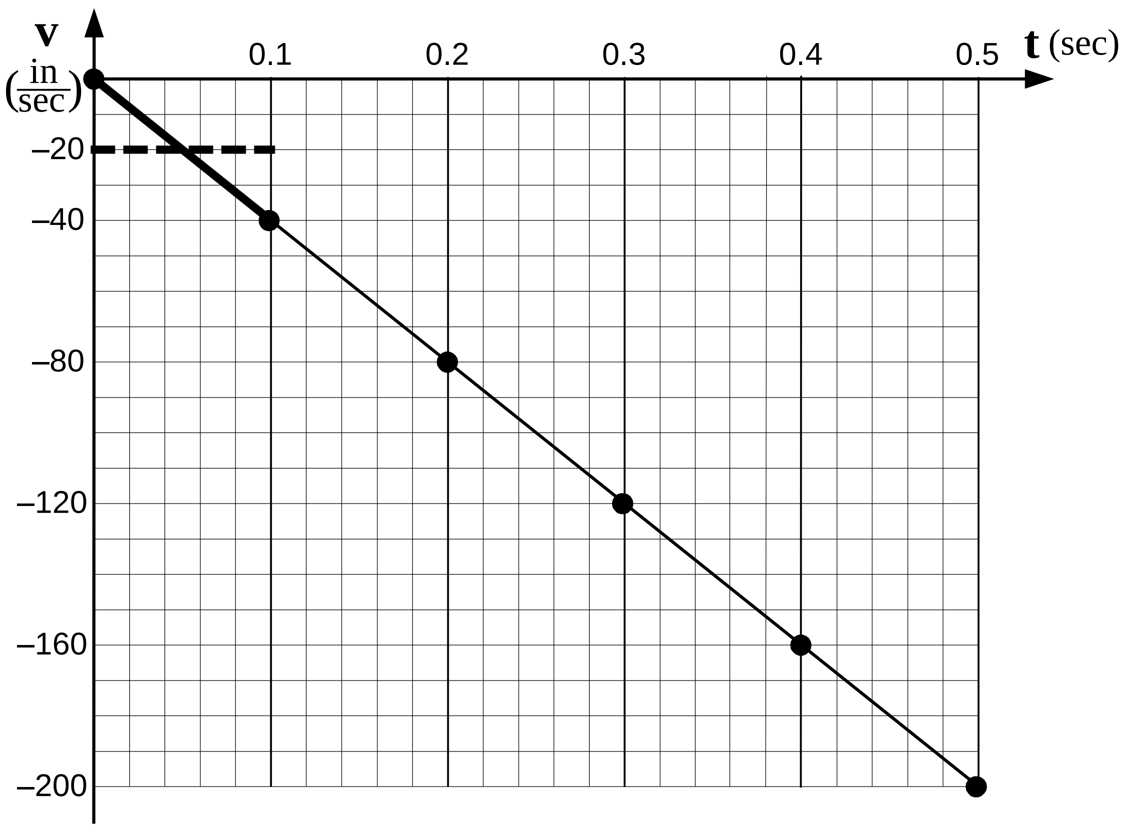
Example 3

We drop a ball that weighs 1 lb; that is, gravity pulls on it with a force of 1 lb. This causes it to accelerate at a rate of –400 (inch/sec)/sec. That is, the ball's downward speed increases by 400 inch/sec for every second that it falls. Actually, a ball of any weight has the same acceleration due to gravity; a heavier ball has more mass, but gravity pulls just that much harder on it. The mass **m** increases, but so does the force **F**. The acceleration due to gravity is

$$\mathbf{a} = -400 \text{ (inch/sec)/sec.}$$

The acceleration is negative (the minus sign) because we choose to say that velocity downward is negative.

Let's plot the velocity of the ball as a function of time. Using the relationship **v** = **a·t** (see Example 1), for the times **t** = 0, 0.1, 0.2, 0.3, 0.4, 0.5 sec we get **v** = 0, –40, –80, –120, –160, –200 inch/sec. We plot these points as below, and connect them with a smooth line.



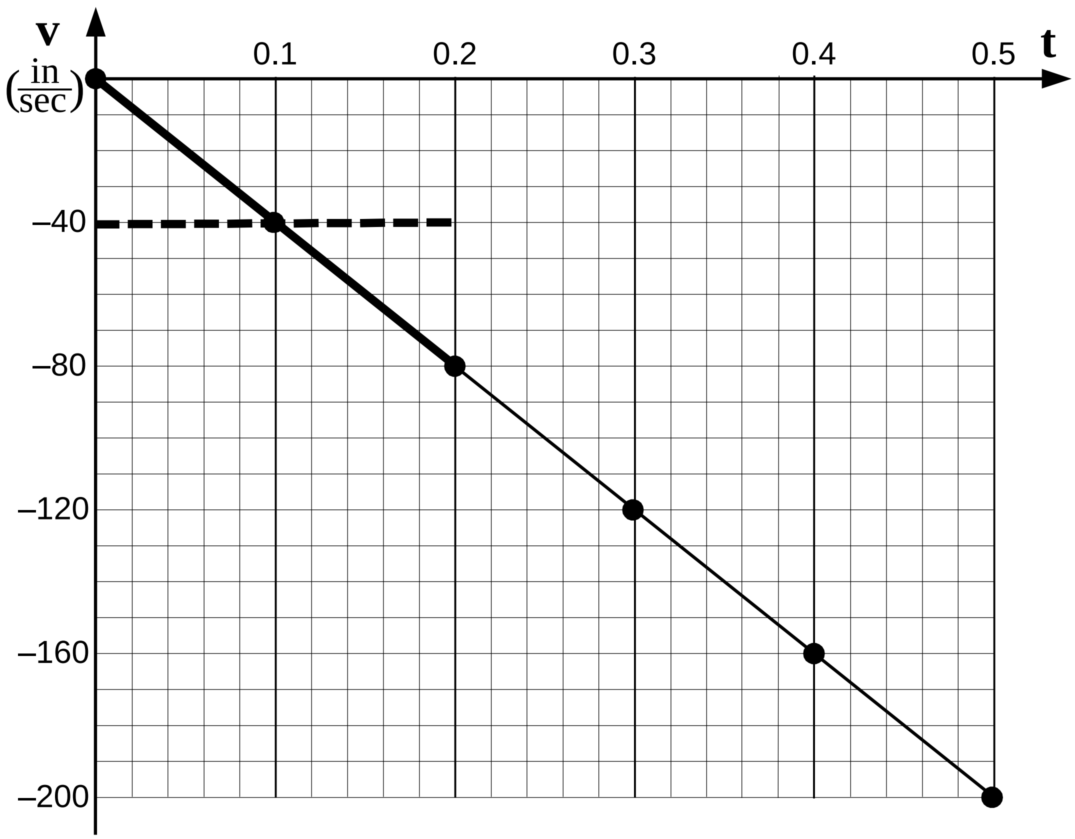
Notice that the plot falls by 40 for each 0.1 sec increase in time, so the slope is

$$\text{slope} = -40/0.1 = -400 \text{ inch/sec},$$

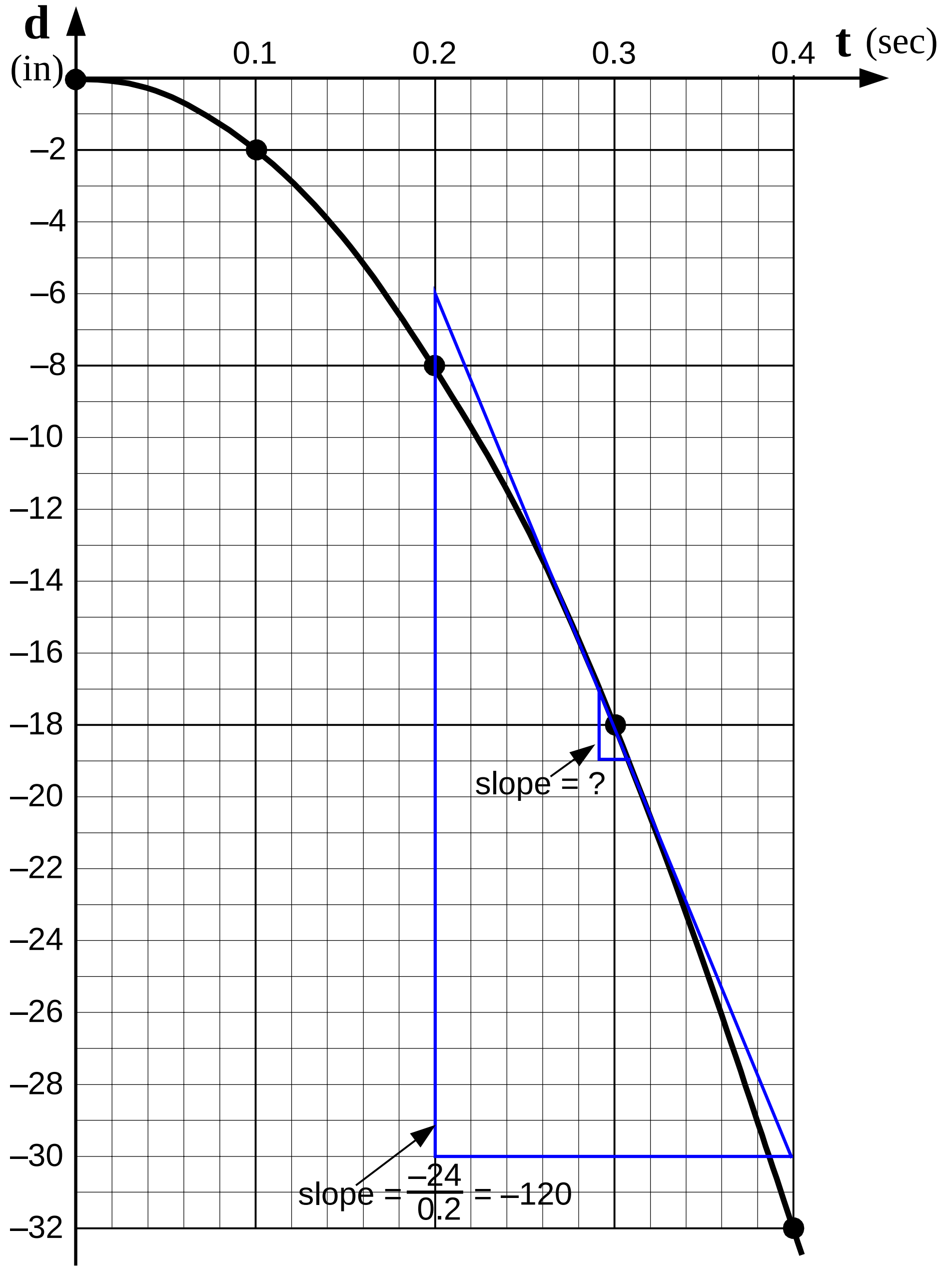
which is the acceleration **a**.

What is the distance the ball falls in the first 0.1 sec? As in Example 1, the distance is the velocity times the time: $\mathbf{d} = \mathbf{v} \cdot \mathbf{t}$. But which value of **v** should we use in the interval from $\mathbf{t} = 0$ to $\mathbf{t} = 0.1$? During that time the velocity went from $\mathbf{v} = 0$ to $\mathbf{v} = -40$ (see the heavy line on the plot on the previous page). So its *average* value during that interval is $\mathbf{v}_{ave} = -20$ inch/sec (see the dashed line on the plot). That is, **v** is as much above –20 as it is below it. Then at the end of that interval (where $\mathbf{t} = 0.1$) we get $\mathbf{d} = \mathbf{v}_{ave} \cdot \mathbf{t} = (-20 \text{ inch/sec}) \cdot (0.1 \text{ sec}) = -2$ inches.

What is the distance the ball falls in the first 0.2 sec? During that time the velocity went from $\mathbf{v} = 0$ to $\mathbf{v} = -80$ (see the heavy line on the plot below). So its *average* value during that interval is $\mathbf{v}_{ave} = -40$ inch/sec (see the dashed line on the plot). Then at the end of that interval (where $\mathbf{t} = 0.2$) we get $\mathbf{d} = \mathbf{v}_{ave} \cdot \mathbf{t} = (-40 \text{ inch/sec}) \cdot (0.2 \text{ sec}) = -8$ inches.



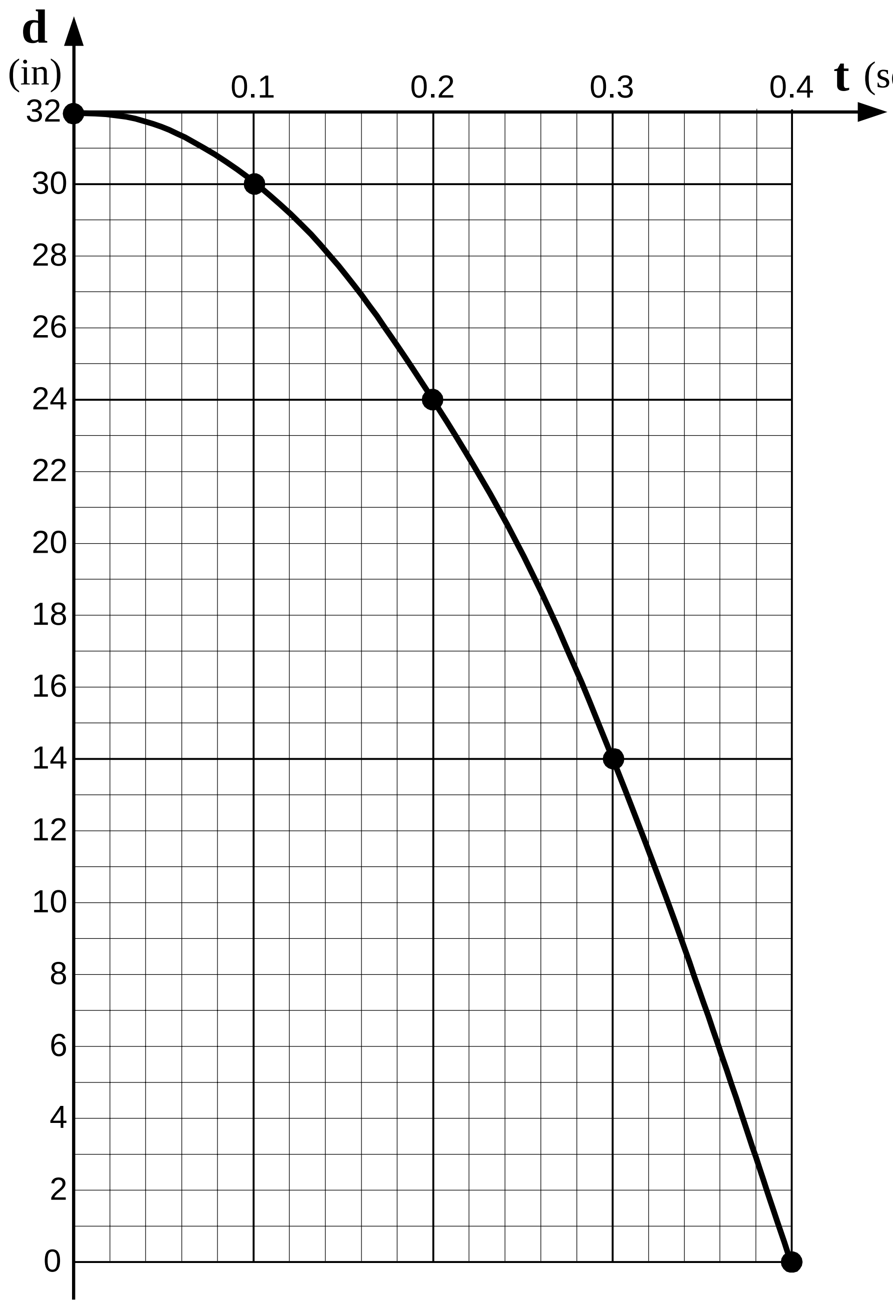
So for $\mathbf{t} = 0.1$ we get $\mathbf{d} = -2$, and for $\mathbf{t} = 0.2$ we get $\mathbf{d} = -8$. We plot these points on the next page, along with some other calculated points. Then we connect the points with a smooth line (a *curve*).



We should be able to test that the slope of this plot for **d** gives us back the velocity **v**. But the plot is not a straight line; the slope keeps changing. So we have to look at a very small interval so the slope doesn't changed much. Lets find the slope near the point for **t** = 0.3 sec. If we take the height of the small blue triangle there and divide by its width, we get the slope. But the height and width are so small that we can't get an accurate measurement. So we lay a ruler against the black plot for **d** at that point and draw the blue line with the same slope as the black plot. Complete the large triangle with a vertical line and a horizontal line. The triangle has a width of 0.4 – 0.2 = 0.2, and its height is –30 – (–6) = –30 + 6 = –24. That is, the slope of the large triangle falls 24 as the time increases by 0.2. Then the slope is slope = (–24 inches)/(0.2 sec) = –120 inch/sec,

which is the same as the velocity **v** for **t**= 0.3 sec (see the plot of **v** on page 4). So even for a curved plot, the slope of the **d** plot (as a function of time **t**) gives us back the velocity **v**.

The ball in this example falls 32 inches in 0.4 sec. We can change our frame of reference (to a different position, not a different speed) so that we call **d** = 32 inches at the top and **d** = 0 at the bottom. We just add 32 to all the numbers on the **d** axis (see the plot below).

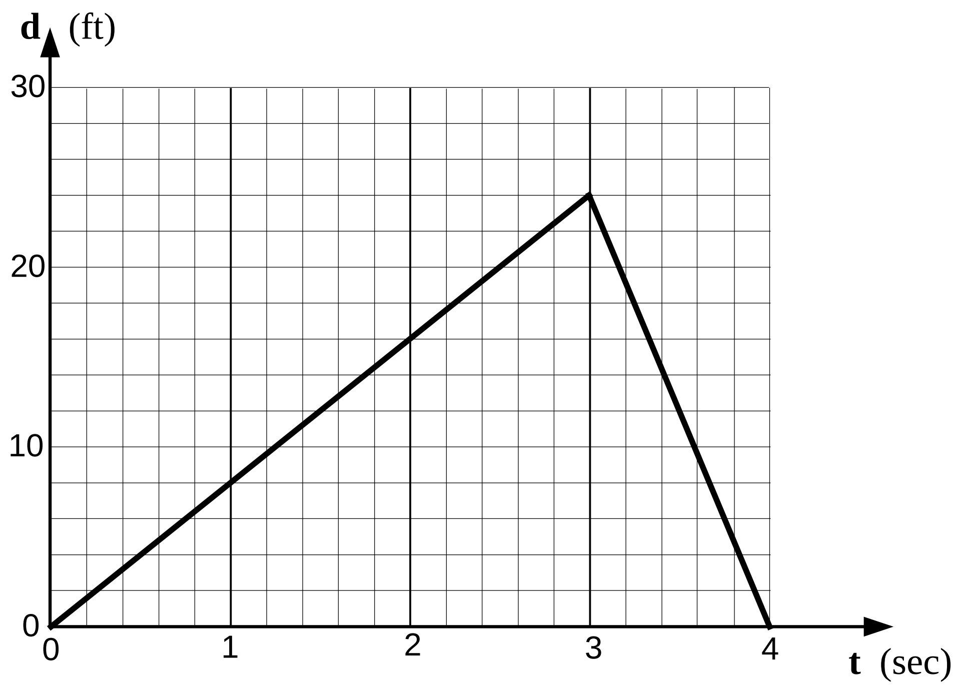


The ball might bounce when it hits the floor at **d** = 0.

Problem 1

A train travels a 60 mph. Its starting point is distance **d** = 0, and its starting time is **t** = 0. Plot its distance as a function of time from **t** = 0 to **t** = 5 hours. Start by plotting points for **t** = 0, 1, 2, 3, 4, and 5 hours. Your plot should label the axes with the symbols for distance and time, with the units, and some values. Find the slope of the plot.

Problem 2



For the plot above, what is each vertical grid worth? What is each horizontal grid worth? Estimate the value of **d** for **t** = 1.6 sec. Find the slope for time between 0 and 3 seconds. Find the slope for time between 3 and 4 seconds. Plot the velocity as a function of time. Label the axes properly.

Problem 3

A car accelerates at 10 mph/sec for 6 seconds. Calculate the velocity for t = 0, 1, 2, 3, 4, 5, and 6 seconds. Plot the velocity from **t** = 0 to **t** = 6 sec. Plot the car's distance for **t** = 0 to **t** = 6 sec.

Problem 4

Calculate **d** in Example 3 for **t** = 0.5 sec. Extend the plot of **d** out to **t** = 0.5 sec. Find the slope of the plot at **t** = 0.4 sec. Does it agree with the value of **v** for **t** = 0.4 sec? (See the plot for **v** in Example 3.)

Problem 5

If we dropped the 1000-lb railroad car in Example 2, gravity would apply a force **F** = 1000 lb on it and cause it to accelerate at **a** = 400 (inch/sec)/sec. Show that this is the same as **a** = 33 (ft/sec)/sec. Using this second expression for **a** and the value for **F**, calculate the mass **m** of the car. The unit for mass in this case is called a *slug*.

Experiment 1

Buy a super ball and measure how high it bounces compared to the distance it was dropped from (it should be about 90%). Drop it from a height so that it takes 0.5 sec to hit the floor (see Problem 4). Measure the time between bounces. Is it about 1 second?

If you can find a place to do it, drop the ball from 16 feet and see if it takes 1 second to hit the floor (2 seconds between bounces). Do the calculation that shows **d** = 16 feet corresponds to **t** = 1 second in Example 3.