Chapter 11
Thermal Energy

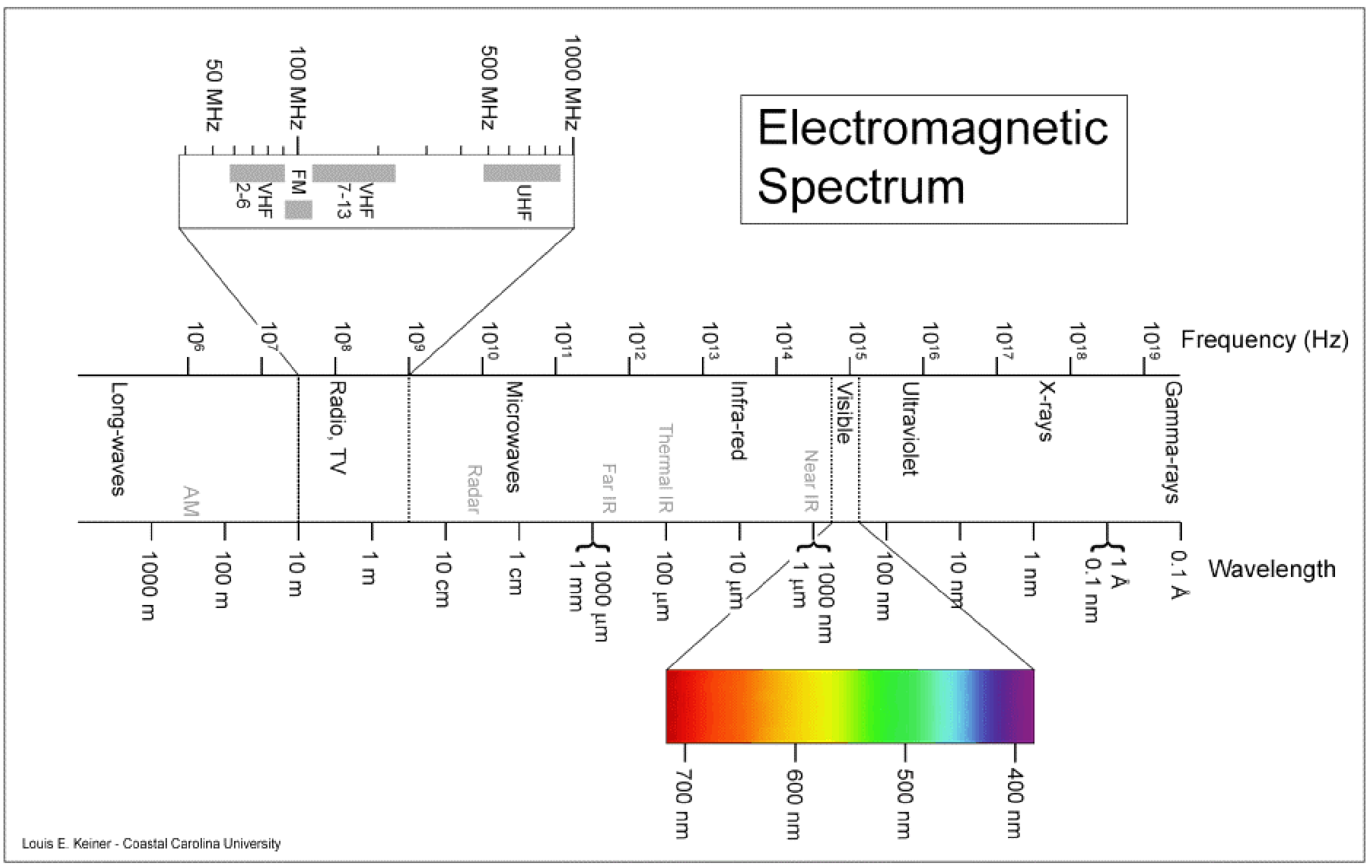
Thermal Energy

Thermal energy is really kinetic energy stored in the motion of molecules and in the vibration of molecules. So far we've used "kinetic energy" to describe the motion of a bunch of molecules all in the same direction and all with the same speed (like in a thrown ball). With thermal energy, each molecule has a different direction and different speed. (This is an example of the "internal energy" in Lesson 4.)

We can get all the energy out of a thrown ball in the form of work when it comes to a stop upon hitting an object and moves that object. There are two ways to get the thermal energy out of something hot: by *conduction* and by *radiation*. By removing some thermal energy from a material we're slowing down the speed and vibration of its molecules. But, in practice, we can never get all of the thermal energy out of it.

Radiation

All heat radiation is electromagnetic, which includes (in decreasing wavelengths) radio waves, microwaves, infrared light, visible light, ultraviolet light, X-rays, gamma rays, and cosmic rays. Shorter wavelengths have high energy and come from a material with higher temperature.



Two examples of hot things losing energy by radiation are the Sun and glowing charcoal in the fireplace. You and a wall each exchange heat by infrared radiation, but you're not aware of it because the heat you lose to the wall equals the heat the wall gives to you. But when you walk by a window, you no longer get the radiation from a warm wall, and you feel like the window is "radiating cold at you."

The rate of heat loss by radiation depends on the surface area **A** and the surface temperature **T** of the object. Here "heat" means thermal energy, which is measured in British thermal units (Btu) or in joules (J), where

$$1 \text{ Btu} = 1055 \text{ J.}$$

The rate of heat loss is thermal power $\mathbf{P}_T$, which is measured in Btu's per second or joules per second. Since one joule per second is one watt (W),

$$1 \text{ Btu/s} = 1055 \text{ J/s} = 1055 \text{ W,}$$

$$3600 \text{ Btu/hr} = 1055 \text{ W,}$$

$$3.41 \text{ Btu/hr} = 1 \text{ W.}$$

We calculate the thermal power radiated by a hot body as follows:

$$\mathbf{P}_T = \mathbf{A} \cdot \mathbf{T}^4 / \mathbf{k}_R$$

where the radiation constant is

$$\mathbf{k}_R = 1{,}990{,}000{,}000 \text{ ft}^2\text{-°Ra}^4\text{/W.}$$

The units of this constant say the equation gives the power in watts (W) if we specify the area in square feet (ft^2) and the temperature in degrees Rankine (°Ra), where 0 °Ra is absolute zero. Absolute zero is the temperature for which there is no thermal energy—no motion (speed or vibration) of molecules. Now, zero degrees Fahrenheit (0 °F) is 460 °Ra, so we have to add 460 to temperature measured in Fahrenheit to get **T** in °Ra.

Note the equation for $\mathbf{P}_T$ has the fourth power of **T**! This means that something twice as hot radiates 16 times as much heat.

Example 1

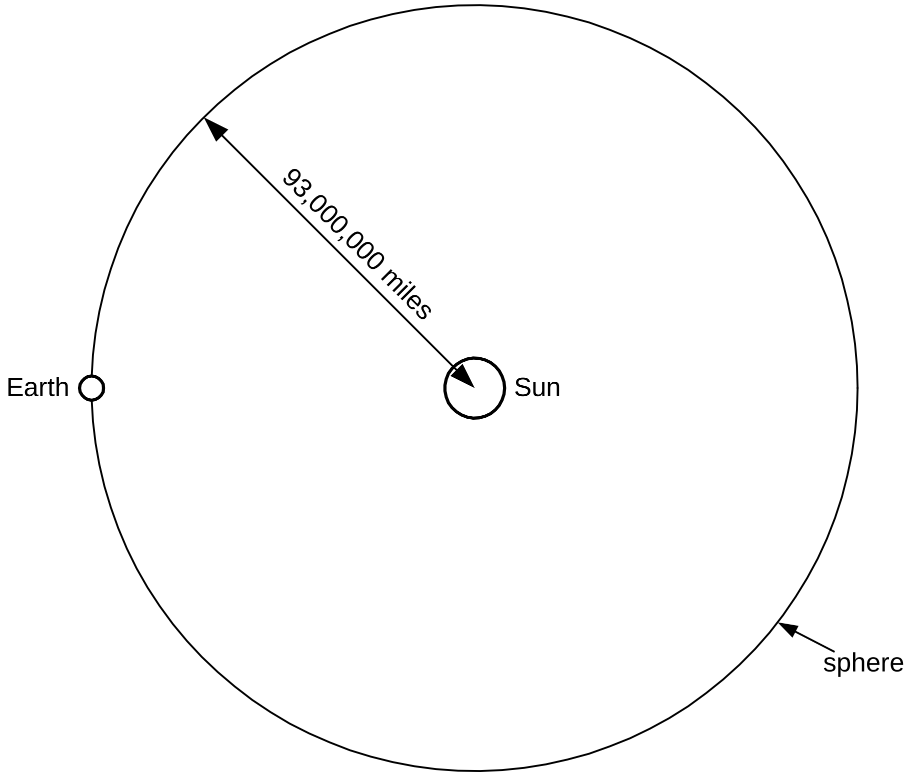
The temperature of the Sun's surface is **T** = 10,360 °Ra, and the area of the Sun's surface is

$$\mathbf{A} = 65{,}500{,}000{,}000{,}000{,}000{,}000 \text{ ft}^2$$

Therefore the thermal power radiated by the Sun is

$$\begin{aligned} \mathbf{P}_T &= \mathbf{A} \cdot \mathbf{T}^4 / \mathbf{k}_R \\ &= (65{,}500{,}000{,}000{,}000{,}000{,}000 \text{ ft}^2) \cdot (10{,}360 \text{ °Ra})^4 / (1{,}990{,}000{,}000 \text{ ft}^2\text{-°Ra}^4\text{/W}) \\ &= 379{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000 \text{ W.} \end{aligned}$$

This is an incomprehensibly large number. But (fortunately for us) not that much of it falls on each square foot of the Earth's atmosphere. The Earth is 93 million miles (491,000,000,000 ft) from the Sun. A sphere with this radius has a surface area of $\mathbf{A}_{sphere}$ = 3,000,000,000,000,000,000,000,000 ft^2, and all the



Sun's power would be distributed evenly over this surface. Then the power falling on each square foot at 93 million miles from the Sun is

$$\text{power density} = \mathbf{P}_T / \mathbf{A}_{sphere}$$

$$= \frac{379{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000 \text{ W}}{3{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000 \text{ ft}^2}$$

$$= 126 \text{ W/ft}^2$$

Therefore the power density of sunlight hitting the Earth's surface is 126 W/ft^2, which equals (126 W/ft^2)·(3.41 Btu/hr/W) = 430 (Btu/hr)/ft^2.

Example 2

A log in the fireplace has turned to charcoal (carbon) and is glowing red with heat. Its temperature is 1200 °F, and its surface area (facing the room) is **A** = 1 ft^2. The temperature of the room is 70 °F. How many Btu's of heat per hour does the log radiate? What is the net radiation (the log's radiation into the room minus the room's radiation into the log)?

The log's Rankine temperature is **T** = 1200 + 460 = 1660 °Ra. Therefore the radiation power is

$$\mathbf{P}_T = \mathbf{A \cdot T}^4/\mathbf{k}_R = (1 \text{ ft}^2)\cdot(1660\ °\text{Ra})^4/(1{,}990{,}000{,}000 \text{ ft}^2\text{-}°\text{Ra}^4/\text{W}) = 3{,}816 \text{ W}.$$

This equals (3,816 W)·(3.41 Btu/hr/W) = 13,000 Btu/hr.

The room's temperature is **T** = 70 + 460 = 530 °Ra. Therefore its radiation onto the 1-ft^2 log surface is

$$\mathbf{P}_T = \mathbf{A \cdot T}^4/\mathbf{k}_R = (1 \text{ ft}^2)\cdot(530\ °\text{Ra})^4/(1{,}990{,}000{,}000 \text{ ft}^2\text{-}°\text{Ra}^4/\text{W}) = 39.6 \text{ W}.$$

This equals (39.6 W)·(3.41 Btu/hr/W) = 135 Btu/hr.

Therefore the net heat radiation from the log is 13,000 – 135 = 12,865 Btu/hr (almost the same as the gross radiation). A 7-lb log stores 50,000 Btu of thermal energy, so it would totally burn to CO_2 in a little less than 4 hours.

Conduction

When you hold a cup of hot cocoa, your hand gets warm as the cardboard (the insulation) of the cup conducts heat from the liquid to your flesh. Whenever two materials are in contact, the material with the more energetic molecules transfers heat energy to the other material as touching molecules shake each other. The more energetic the molecules, the higher the temperature, and heat always flows from the higher temperature to the lower temperature. (The relation between thermal energy in an object and temperature of the object depends on the *specific heat* of the material.)

The thermal power $\mathbf{P}_T$ transferred by conduction depends on the area **A** of contact, the temperature difference $\Delta\mathbf{T}$ between the two materials, and the total thermal *resistivity* $\mathbf{R}_{total}$ of the insulation between the materials:

$$\mathbf{P}_T = (\Delta\mathbf{T}\cdot\mathbf{A})/\mathbf{R}_{total}$$

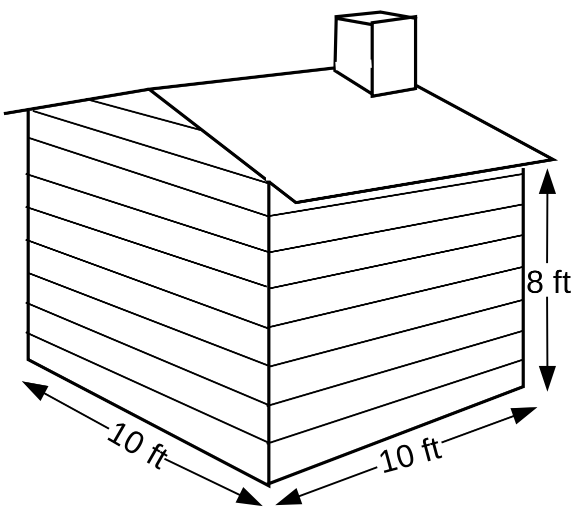
This is like the relationship in Lesson 10 in which the rate of flow **f** of water depends on the pressure difference **p** and the hydraulic resistance **R**:

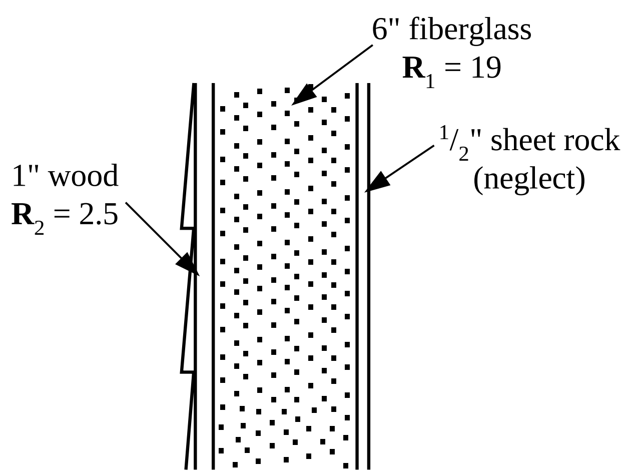
$$\mathbf{f} = \mathbf{p}/\mathbf{R}.$$

The thermal energy flow rate is $\mathbf{P}_T$, the thermal “pressure” is $\Delta\mathbf{T}$, and the thermal resistance is $\mathbf{R}_{total}/\mathbf{A}$.

Example 3

A 10-ft ×10-ft cabin has $\mathbf{A}$ = 430 ft^2 exposed to the weather. A stove keeps the inside at $\mathbf{T}_1$ = 70°F both day and night while the outside temperature is $\mathbf{T}_2$ = 20 °F both day and night. The walls and roof have ½ inch of sheet rock, 6 inches of fiberglass insulation, and 1 inch of wood sheathing. What is the rate of heat loss? How many days will a cord of wood heat the house?





The thermal resistivity of fiberglass is $\mathbf{R}_1 = \mathbf{d}_1 \times 3.2$ (ft^2-°F-hr/Btu)/inch, where $\mathbf{d}_1$ is the thickness in inches. Since $\mathbf{d}_1$ = 6 inches, $\mathbf{R}_1$ = (6 in) × 3.2 (ft^2-°F-hr/Btu)/inch = 19 ft^2-°F-hr/Btu.

The thermal resistivity of wood is $\mathbf{R}_2 = \mathbf{d}_2 \times 2.5$ (ft^2-°F-hr/Btu)/inch, where $\mathbf{d}_2$ is the thickness in inches. Since $\mathbf{d}_2 = 1$ inch, $\mathbf{R}_2 = (1 \text{ in}) \times 3.2$ (ft^2-°F-hr/Btu)/inch = 2.5 ft^2-°F-hr/Btu.

The sheet rock has negligible insulating effect. Therefore the total thermal resistivity is $\mathbf{R}_{total} = \mathbf{R}_1 + \mathbf{R}_2 = 19 + 2.5 = 21.5$ ft^2-°F-hr/Btu.

The temperature difference is $\Delta\mathbf{T} = \mathbf{T}_1 - \mathbf{T}_2 = 70 - 20 = 50$ °F. The rate of heat loss (thermal power) through the walls and roof is

$$\mathbf{P}_T = (\Delta\mathbf{T}\cdot\mathbf{A})/\mathbf{R}_{total} = (50\ °F)(430\ ft^2)/(21.5\ ft^2\text{-}°F\text{-}hr/Btu) = 1000\ Btu/hr.$$

(We know the units of $\mathbf{P}_T$ are Btu/hr because the units of resistivity are ft^2-°F-hr/Btu.) So the stove must produce 24,000 Btu's per day. One cord of hardwood has 20,000,000 Btu's, but only 60% of this heats the cabin; the other 40% goes up the chimney. So the net thermal energy from a cord is $\mathbf{E}_T$ = 0.60 × 20,000,000 = 12,000,000 Btu. Then one cord will last

$$\mathbf{t} = \mathbf{E}_T / \mathbf{P}_T = (12{,}000{,}000\ Btu)/(24{,}000\ Btu/day) = 500\ days.$$

This is optimistic since we have neglected heat loss through the window glass and neglected cold air coming in through cracks and when someone opens the door.

Here is the thermal resistivity for some materials:

Wood: $\mathbf{R} = \mathbf{d} \times 2.5$ (ft^2-°F-hr/Btu)/inch
Fiberglass: $\mathbf{R} = \mathbf{d} \times 3.2$ (ft^2-°F-hr/Btu)/inch
Polystyrene: $\mathbf{R} = \mathbf{d} \times 5.0$ (ft^2-°F-hr/Btu)/inch
Polyurethane $\mathbf{R} = \mathbf{d} \times 6.8$ (ft^2-°F-hr/Btu)/inch

Fuel Cost

What heating fuel gives you the most energy per dollar? The table below gives the number of Btu's in a quantity of fuel, such as a cord or a ton, depending on how the fuel is sold. It also gives the price of that quantity (as of July, 2008). The gross energy per dollar is given by

$$\text{gross energy/\$} = \text{energy/price},$$

and the net energy (accounting for heat lost up the chimney) is given by

$$\text{net energy/\$} = \text{gross} \times \text{eff}/100$$

The fuels are arranged so those with the most net Btu/$ are at the top.

fuel	energy		price *		gross		eff.%	net	
Heat Pump (COP = 3.8)	13,000	Btu/kWh	0.168	$/kWh	77,381	Btu/$	100	77381	Btu/$
Natural Gas	1,029,000	Btu/1000 ft^3	11.00	$/1000 ft^3	93,545	Btu/$	78	72965	Btu/$
Coal	26,000,000	Btu/ton	290	$/ton	89,655	Btu/$	75	67241	Btu/$
Cord Wood (hard)	20,000,000	Btu/cord	180	$/cord	111,111	Btu/$	60	66667	Btu/$
Wood Pellets	17,000,000	Btu/ton	250	$/ton	68,000	Btu/$	80	54400	Btu/$
Propane - nonvented	91,600	Btu/gal	3.00	$/gal	30,533	Btu/$	100	30533	Btu/$
Fuel Oil (#2)	139,000	Btu/gal	4.50	$/gal	30,889	Btu/$	78	24093	Btu/$
Electricity -water heater	3412	Btu/kWh	0.142	$/kWh	24,028	Btu/$	100	24028	Btu/$
Propane -vented	91,600	Btu/gal	3.00	$/gal	30,533	Btu/$	78	23816	Btu/$
Electricity	3412	Btu/kWh	0.168	$/kWh	20,310	Btu/$	100	20310	Btu/$

*Prices as of July 2008

Example 4

A house needs 60,000,000 Btu of net thermal energy per winter. How much would it save in fuel costs per year if it converted from oil to wood pellets? (This assumes fuel prices as of July 2008.)

The net energy per dollar for oil is 24093 Btu/$, so the cost for the winter is

$$(60{,}000{,}000 \text{ Btu})/(24093 \text{ Btu/\$}) = \$2490.$$

The net energy per dollar for wood pellets is 54400 Btu/$, so the cost for the winter is

$$(60{,}000{,}000 \text{ Btu})/(54400 \text{ Btu/\$}) = \$1103.$$

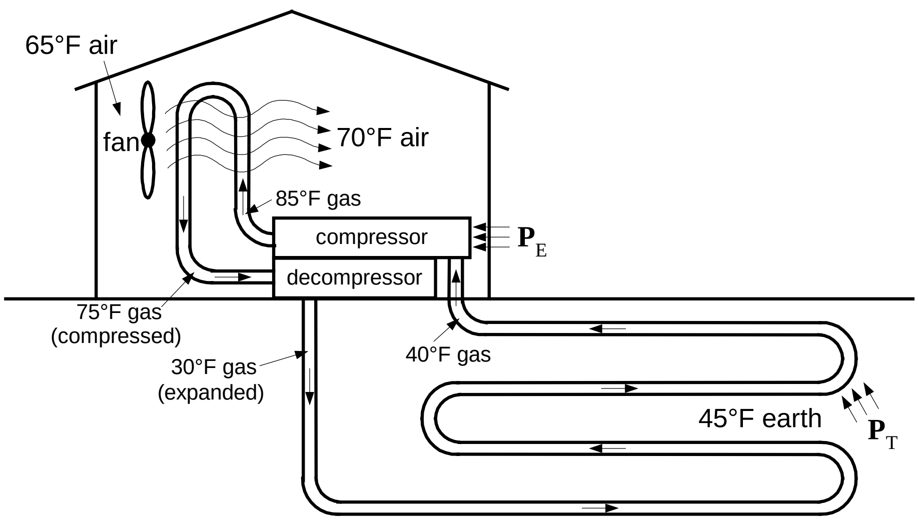
So the saving is $2490 – $1103 = $1387/yr. If the wood-pellet stove, flue pipe, and installation costs are $2700, the new heating system pays for itself in two years.

Heat Pumps

The simplest way to get heat from electric power is to run current through a resistor in a space heater (see Lesson 10, Example 4). The heat power equals the electrical power; the process is 100% efficient since no power goes up a chimney.

$$\mathbf{P}_T = \mathbf{P}_E = \mathbf{v}\cdot\mathbf{i}.$$

A more efficient way to use the electrical power is to run a heat pump (compressor and decompressor) with the $\mathbf{P}_E = \mathbf{v}\cdot\mathbf{i}$ and pump heat from a lower temperature (usually the ground) to a higher temperature—the inside of the house. The diagram below shows the arrangement. A hot (85°F) compressed gas passes



through coils in the house with a fan blowing air through the coils. The air starts out at 65°F but warms to 70°F after passing through the coils. In giving up heat to the air, the compressed gas has cooled to 75°F. Then the gas passes through the decompressor where the expansion of the gas cools it to 30°F. The expanded gas passes through the 45-°F earth and warms to 40°F, taking heat from the ground. The compressor then raises the gas to 85°F again, and the heat taken from the ground is given to the air in the house.

We wish that heat from the 45-°F earth would simply flow by itself into the 70-°F house. But heat can't flow from a low temperature to a higher temperature; it needs the help of the heat pump. By compressing the 40-°F gas it raises to 85°F the heat $\mathbf{P}_T$ which the gas has "absorbed" from the earth, and it also adds the electrical power $\mathbf{P}_E = \mathbf{v} \cdot \mathbf{i}$ (from running the heat pump) to the gas. So the total thermal power given to the house is

$$\mathbf{P}_{total} = \mathbf{P}_T + \mathbf{P}_E.$$

How much electrical power $\mathbf{P}_E$ is needed to raise a certain amount of thermal power $\mathbf{P}_T$ from $\mathbf{T}_1$ = 40 °F (the temperature of the gas going into the compressor) up to $\mathbf{T}_2$ = 85 °F (the temperature of the gas leaving the compressor)? It is proportional to both $\mathbf{P}_T$ and the temperature difference $\Delta\mathbf{T} = \mathbf{T}_2 - \mathbf{T}_1$.

$$\mathbf{P}_E = \mathbf{P}_T \cdot \Delta\mathbf{T}/\mathbf{k}_{hp},$$

or

$$\mathbf{P}_T = \mathbf{P}_E \cdot \mathbf{k}_{hp}/\Delta\mathbf{T},$$

where $\mathbf{k}_{hp}$ is a constant that depends on the design of the heat pump. Typically $\mathbf{k}_{hp}$ = 126 °F. Then

$$\mathbf{P}_{total} = \mathbf{P}_T + \mathbf{P}_E = \mathbf{P}_E \cdot \mathbf{k}_{hp}/\Delta\mathbf{T} + \mathbf{P}_E$$

$$= \mathbf{P}_E(\mathbf{k}_{hp}/\Delta\mathbf{T} + 1) = \mathbf{P}_E \cdot \mathrm{COP},$$

where

$$\mathrm{COP} = \mathbf{k}_{hp}/\Delta\mathbf{T} + 1$$

is the "Coefficient Of Performance." For $\Delta\mathbf{T}$ = 85 – 40 = 45°F and $\mathbf{k}_{hp}$ = 126 °F, then COP = 126/45 + 1 = 3.8. That is, we get 3.8 times more heat from the electrical power $\mathbf{P}_E$ put into a heat pump than we do from the same $\mathbf{P}_E$ put into a space heater.

Note that the COP is less if we have to raise the temperature of the gas by a larger $\Delta\mathbf{T}$. If the coils in the house were raised to 220°F (like a steam radiator is), then $\Delta\mathbf{T}$ = 220 – 40 = 180, and we would get a factor of only COP = 126/180 + 1 = 1.7. So low-temperature coils in the house (like those used for forced hot air and for radiant heating) are best.

Problems

Problem 1

Suppose we put a skylight in the south-facing slope of the room of the cabin in Example 3. This will allow solar radiation to warm the interior. Half of the solar radiation that comes in is lost by re-radiation back up through the window and from heat-conduction losses due to the poor R-value of the window. What area of skylight is needed to keep the cabin at 70°F when it is 20° outside? Assume the sun effectively shines for only 8 hours a day (this accounts for times when it isn't shining straight in the skylight).

Problem 2

The log in Example 2 is deprived of air, and it stops burning. After a while it cools to 70°F. Find its net radiation now.

Problem 3

In Example 3 the stove must produce 24,000 Btu's per day. We could reduce this by letting the temperature in the cabin go down to 60°F at night (from 7:00 p.m. to 7:00 a.m.). How many Btu's per day would the stove have to produce then? (It's still 70°F in the cabin during the day.)

Problem 4

A house needs 60,000,000 Btu of net thermal energy per year. What is the cost if it uses electric power with space heaters? (Use July 2008 prices.) What is the cost saving if it converts to a heat pump with COP = 3.8? (Use July 2008 prices.)

Problem 5

The heat pump in Example 4 could still work in theory if the gas temperature into the compressor were $\mathbf{T}_1$ = 44°F and the gas temperature out of the compressor were $\mathbf{T}_2$ = 71°F. Then heat would still flow by conduction from the 45-°F earth into the 44-°F gas, and heat would still flow from the 71-°F gas into the 70-°F air. But with such a small temperature difference (1°F) the heat-transfer rate $\mathbf{P}_T$ would be too slow unless the surface area **A** of the coils were increased (see **Conduction** in this Lesson). Suppose we use longer coils and $\mathbf{T}_1$ = 44°F and $\mathbf{T}_2$ = 71°F. Find the new COP for the heat pump. (The heat pump constant is still $\mathbf{k}_{hp}$ = 126 °F.)